NO. HY5
OCTOBER 1956

JOURNAL of the

Hydraulics Division

PROCEEDINGS OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOLUME 82

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Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

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Papers

Journal of the
HYDRAULICS DIVISION
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FLOOD PROTECTION OF CANALS BY LATERAL SPILLWAYS

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(Proc. Paper 1077)

SYNOPSIS

To avoid topping of the banks of canals during flood, lateral spillways are frequently used. However, a lateral spillway by itself is not capable of reducing the height of the excessive energy head. The energy reduction has to be accomplished upstream from the spillway.

It has been proven that the water level in front of a lateral spillway crest can be computed by the Bernouilli and the continuity equations. No trials are required in computing the action of a lateral spillway of uniform section, when the friction effect is negligibly small, or the bottom slope is made equal to the average friction slope.

A brief survey is given about different water surface profiles upstream, in front, and downstream from the spillway, depending on the flow stage and on the location of the spillway in the canal.

The effect of non-uniform velocity distribution is taken into account when computing the surface profiles in front of the spillway crest.

To demonstrate the effectiveness of lateral spillways, several spillway layouts with different sections and with additional orifices are analyzed. Their performances are illustrated by depth-flow curves. The conditions maintaining the subcritical flow stage in lateral spillway section are discussed. Further, the efficiency of the combined action of the intake and lateral spillway in raising the headwater during floods is investigated.

Notation

The letter symbols introduced in this paper are defined where they first appear, in the text, or by illustration, and are assembled alphabetically in the Appendix for convenience of reference.

Note: Discussion open until March 1, 1957. Paper 1077 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 82, No. HY 5, October, 1956.

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A. Hydraulics of Lateral Spillway

1. Introduction

Using an open canal for diversion from a reservoir, the canal depth required for normal flow has to be increased so that it may convey the maximum inflow during the floodwater without overflowing its banks.

It is economical to keep this freebord of a diversion canal as low as possible, particularly, if the canal is of substantial length. This is possible by using a lateral spillway which provides the outlet for surplus water.

Even, when the diversion inflow is regulated by a float or remote controlled gate, the installation of a lateral spillway is necessary. Such a control is not absolutely reliable because of the possible mechanical or power failure.

Further, a lateral spillway is necessary at the end of a canal to discharge the full flow at the abrupt closure of the turbines and along a long canal to spill the excess inflow from the hillside.

The first publications of the laboratory tests about the performance of lateral spillways by H. Engels[2] and G. S. Coleman and D. Smith[3] were rathe confusing for hydraulic engineers. The tests by Engels indicated a rising and those by Coleman and Smith a dropping water surface along the spillway crest. This contradiction was caused by the different stages of the flow used subcritical in Engels' and supercritical in Coleman-Smith's tests.

Ph. Forchheimer[4] considered the energy level to be parallel to the spillway crest and to the canal bottom and assumed the water surface profile along the spillway crest to be a straight line.

G. De Marchi[5] proved theoretically that the energy head along the lateral spillway is essentially constant and the water surface profile curved: rising in subcritical and dropping in supercritical flow stages. Further, De Marchi introduced a graphical method for designing lateral spillways.

Br. Gentilini[6] verified De Marchi's theory experimentally.

Jos. Frank's[7] publication concerned with the reciprocal action of the intake gate and the lateral spillway.

O. Herz[8] and H. Favre[9,10] established general methods for stepwise com putations of the water surface in the lateral spillways.

2. "Mitteilungen aus dem Dresdner Flussbaulaboratorium," a/Forschungsa beiten des Vereins Deutscher Ing., Vol. 200/201, 1917. b/Zeitschrift des Vereins Deutscher Ingenieure, 1918, pp. 362, 387, and 412, 1920, p. 101.
3. "Discharging-Capacity of Side Weirs." Institution of Civil Engineers, V No. 6, 1923, London.
4. "Grundriss der Hydraulik," 1926, p. 406, Leipzig-Berlin.
5. "Saggio di teoria del funzionamente degli stramazzi laterali (o sfiaratori longitudinali), "L'Energia Elettrica, 1934, Vol. 11.
6. "Ricerche sperimentali sogli sfioratori longitudinali, "L'Energia Elettrica, 1938, Vol. 9.
7. "Sicherung der Werkkanäle gegen eindringendes Hochwasser," Wasserkraft und Wasserwirtschaft, 1941, Vol. 1, p. 12.
8. "Beitrag zur Abfuhrberechnung von Streichwehre," Wasserkraft, 1925, pp 41, 59, and 99.
9. "Contribution à l'étude des courants liquides," Zürich, 1933.
10. "Expériences sur le mouvement permanent de l'eau dans les canaux découverts, avec apport ou prélèvement de long du courant," Bulletin technique de la Suisse Romande, 1937.

W. H. R. Nimmo[11] applied momentum equation to compute the water level variations in front of the lateral spillway crest.

2. Nature of Flow Along the Lateral Spillway

Fundamentally, the flow along the crest of a lateral spillway may be treated as a flow division where the divided water, lost over the crest of the spillway, does not affect the energy head. The same conclusion may be drawn when the flow in the spillway section is regarded as an imaginary expansion. Thus, the Bernouilli equation combined with the continuity equation is applicable in computing the lateral spillway action.

The variation of the water depth, Δy, along a spillway section of the length, Δx, may be computed easily by the cut-and-try method, using Eq. 1,

$$\Delta y + (S_f - S)\Delta x = C_v \frac{V_1^2 - V_2^2}{2g} \tag{1}$$

in which

$$V_2 = \frac{Q_1 - C\, h_m^{3/2} \Delta x}{A_2} \tag{2}$$

In Eqs. 1 and 2, S and S_f are bottom and friction slopes, respectively; V_1 and V_2 are the corresponding mean velocities in the upstream and in the downstream end of the section; g is the gravitational acceleration; Q_1 represents the rate of flow in the upstream end of the section; h_m is the average head on the spillway crest in the section; A_2 is the flow area in the downstream end of the section; C is the discharge coefficient of the spillway; and C_v is the coefficient of the velocity distribution (Coriolis).

Eq. 1, friction term S_f equalized to the bottom slope S, C_v assumed to be unity, and V_2 replaced by its value (Eq. 2), can also be derivated from the momentum loss in the section. The momentum in the upstream and in the downstream ends of the section, Δx, are:

$$M_1 = \frac{w}{g} Q_1 V_1 \quad \text{and}$$

$$M_2 = \frac{w}{g}(Q_1 - C\, h_m^{3/2} \Delta x)\, V_2 \quad ,$$

in which w is the unit weight of water. Replacing V_2 by $V_1 - \Delta V$, the momentum difference in both section ends, $\Delta M_1 = M_1 - M_2$, can be expressed as follows:

$$\Delta M_1 = \frac{w}{g}\left\{ Q_1 \Delta V - C\, h_m^{3/2} \Delta x (V_1 - \Delta V) \right\} \tag{3}$$

To this momentum difference, the momentum lost by the discharge over the spillway crest:

11. "Side Spillway for Regulating Diversion Canals," ASCE Transactions, Vol. No. 92, 1928, p. 1561.

$$\frac{w}{g}\, C\, h^{3/2}\, \Delta x \left(V_1 - \frac{\Delta V}{2}\right) \tag{4}$$

has to be added. Thus the total momentum change, ΔM, in a lateral spillway section is expressed as follows:

$$\Delta M = \frac{w}{g}\left(Q_1 \Delta V - C\, h_m^{3/2} \frac{\Delta V}{2} \Delta x\right) \tag{5}$$

This corrected momentum loss along a section of a lateral spillway, modified and equated with the decelerating force in the section in the same way as done by Julian Hinds,[12] yields the equation:

$$\Delta y = \frac{Q_1}{g} \frac{V_1 + V_2}{Q_1 + Q_2} \Delta V \left(1 - \frac{C\, h_m^{3/2}\, \Delta x}{2Q_1}\right) \tag{6}$$

Eq. 6 is fully identical with Eq. 1 as can be easily shown by replacing ΔV with $V_1 - V_2$ and $C\, h_m^{3/2}\, \Delta x$ with $Q_1 - Q_2$, when in Eq. 1 the friction factor is omitted and C_v is unity.

3. Diversity of Flow Surface Profiles

The water surface profiles upstream, in front, and downstream from a spillway depend on the flow stage, on the relative height of the spillway crest, on the velocity distribution in the cross-section, and on the location of the spillway in the canal. In case of subcritical flow in a canal of mild slope, the water depth at the downstream end of the spillway is determined by the conveyance depth in the canal or by the backwater from a control structure or from a close forebay. The depth at the upstream end of the spillway is controlled by the constant energy head along the spillway. Consequently, the surface profile upstream from the spillway has to be a straight or either a drop-down or a backwater curve, depending on the necessity to maintain, to absorb, or to preserve the energy in the approaching flow so that it would match the energy head downstream from the spillway.

Following characteristic variations of the surface profiles may be distinguished:

a. The lateral spillway located close to the intake gate, Fig. 1a. Its purpose is to spill off the excessive rate of the diversion flow during the high head water. The energy loss of the flow, passing through the gate opening, should absorb the excess energy head between the flood level in the reservoir and the maximum permissible conveyance energy head in the canal. The water level between the gate and the spillway is affected by the unequal velocity distribution behind the submerged gate opening and in the spillway section.
b. The spillway is located in the middle of a long canal to skim off the excessive surface or the side-stream inflow, Fig. 1b. The water surface

12. "Side Channel Spillway," ASCE Transactions, Vol. 87, 1926, p. 894.

downstream from the spillway is controlled by the maximum permissible conveyance depth as in the preceding case. As the energy head in the upstream flow is higher than downstream, the approaching flow follows a dropdown curve in order to destroy the excessive energy by friction. Of course, enough canal length on the upstream side is required to develop the necessary down-drop; otherwise, the spillway would not be as effective as intended.

. The spillway is located at the end of a canal close to the forebay, for the purpose of discharging the full inflow at abrupt shutdown of the plant, Fig. 1c. (The forebay is assumed to be large enough to be able to dampen the closure surge). In this case, the water level at the end of the lateral spillway is higher than the conveyance head in the conduit. To match the higher energy head, the water level upstream from the spillway builds up a backwater profile to obtain the necessary rise of energy head.

. The flow in a canal of subcritical slope may pass into the supercritical stage at the beginning of the spillway and jump up again to the subcritical stage downstream, Fig. 1d. Such undesired flow may occur:

1. When the spillway crest is below the critical depth.(Prof. A. Schocklitsch[13]), or
2. When the ratio of the velocity upstream from the lateral spillway to the critical velocity exceeds 0.75 (M. Schmidt[14]). Under such flow conditions, the profile of the water surface and the performance of the spillway cannot be determined without model tests. The jump may occur already in front of the crest. Further, the spillway capacity cannot be increased much by lengthening its crest because the hydraulic jump is shifted downstream when the spillway is made longer.

. In a canal of supercritical slope, the water surface drops in front of the spillway crest, Fig. 1c. With increase of the velocity, the energy level in the spillway section cannot be held parallel to the bottom because of a strong increase of friction loss as observed by Gentilini.[6]

. Cross-Sectional Velocity Distribution

As shown in paragraph A-2, the water surface in front of a lateral spillway an be computed by Bernouilli equation, imagining that the flow section is idened correspondingly to the flow division over the spillway crest.

Actually, only the flow section above the spillway crest can expand freely. onsequently, an unequal velocity distribution will ensue in the spillway secon. This condition may be aggravated by the unequal velocity distribution ehind the submerged gate opening.

For the computation of the water level, a velocity coefficient, C_v, is introiced to take care of the depression of the water level caused by the unequal ross-sectional velocity distribution. The magnitude of such velocity coefcients could be established by model tests or by prototype measurements. ortunately, two papers concerning the surface depression in the lateral illway flow have been recently published by Dr. Engs. M. Schmidt[14] and

. "Handbuch des Wasserbaues," Julius Springer, Vienna, 1950, Vol. 1, p. 136.

. "Die Berechnung von Streichwehren," Die Wasserwirtschaft, Vol. 1, 1955, p. 96.

Zschiersche:[15] C_y-values of the magnitude up to 1.3 at the beginning of the spillway have been established. Even higher values have been found at the end of the spillway crest.

According to Schmidt, the lateral spillway flow is not controlled completely by the Bernouilli equation. To adjust the Bernouilli equation for the lateral spillway, Schmidt introduces a ratio, C_y, for the variation of the depth to the variation of the kinetic energy along the spillway:

$$C_y = \frac{\bar{y}_2 - \bar{y}_1}{1.1\,\frac{V_1^2}{2g} - 1.1\,\frac{V_2^2}{2g} - L\,S_f} \qquad (7)$$

$\bar{y}_1$ and $\bar{y}_2$ are the respective actual depths in up- and down stream ends of the spillway; V_1 and V_2 are the corresponding mean velocities; and LS_f is the friction loss. In Fig. 2a, the values of C_y for two different lateral spillway layouts, obtained from experimental study by Schmidt, are plotted as functions of the ratio, $\frac{h_m}{z + h_m}$, in which z is the height of the spillway crest. The curve-(Q_2>0) represents the C_y-values for the regular lateral spillways behind intake structure as described in case a in paragraph A 3. The curve-(Q_2=0) is applicable for the lateral spillways located at the ends of the conduits (case c, paragraph A 3). Zschiersche, conducting similar tests, established correction coefficients for empirical use.

As described previously, the flow over the lateral spillway crest is a flow division. Fundamentally, no appreciable change in the energy level can occur in a flow division. The higher friction loss due to the increased nonuniformity of the cross-sectional velocity distribution is taken care of by multiplying the conventional friction loss, LS_f, by the coefficient C_y. As mentioned above, the friction loss can be compensated by the corresponding bottom drop. Consequently, the simplified Bernouilli equation:

$$\bar{y}_1 + C_v \frac{V_1^2}{2g} = \bar{y}_2 + C_v \frac{V_2^2}{2g} = H_s \qquad (8)$$

is applicable for the computations of the lateral spillway flow. In Eq. 8, C_v is 1.1 times the respective C_y-value obtained from the graph in Fig. 2a and H_s is the constant energy head in the spillway section. Actually, the velocity coefficient C_v varies along the spillway crest (Schmidt). As there is no experimental data about its variation available, the value of C_v is assumed to be constant along the crest of a particular spillway. For conveniency, the C_v-values are plotted as functions of the ratio, $\frac{h_m}{z + h_m}$, in Fig. 2b.

15. "Die Berechnung von Streichwehren auf Grund von Modellversuchen geraden und schräg gestellten Streichwehren, "Veröffentlichung der Forschungsanstalt für Schiffahrt, Wasser- und Grundbau No 2, Akademie Verlag, Berlin, 1954.

5. Discharge Coefficient of Lateral Spillway

According to the experimental studies of the action of lateral spillways by Gentilini,[6] the discharge coefficients established by Bazin for regular sharp-crested weirs may be also used for lateral spillway crests, if reduced 7 per cent:

$$Q = C\ L\ H_m^{3/2} \qquad \text{in which} \tag{9}$$

$$C = 0.93(3.248 + \frac{0.079}{h_2})(1 + 0.55\ \frac{h_2^2}{\bar{y}_2^2}) \tag{9a}$$

valid in limits: $\bar{y}_2/H_s > 0.9$ and $z/H_s > 0.75$. Forcheimer[4] and Schmidt[14] proposed to use for lateral spillways of any crest shape the corresponding discharge coefficients of the regular weirs reduced by 5 per cent.

B. Lateral Spillway at Conduit Intake

1. Performance of Simple Lateral Spillway

The writer investigates the performance of the most common lateral spillway located behind the intake structure (Fig. 3a). The purpose of this spillway layout is to protect the canal banks against over flooding during the water level rise in the reservoir. The spillway crest is placed so high that no water is wasted under normal flow conditions.

In a uniform spillway section, the water depths can be computed using the Bernouilli equation:

$$\bar{y} + C_v\ \frac{V^2}{2g} + (S_f - S)\Delta x = H_s \tag{10}$$

in which the mean velocity, V, is reduced in the downstream direction because of the loss of water over the spillway. Replacing V with its value $\frac{Q}{\bar{y}}$ and equating the friction slope S_f to the bottom slope S, Eq. 10 for a uniform rectangular section is modified to:

$$Q = b\sqrt{\frac{2g}{C_v}}\sqrt{H_s\bar{y}^2 - \bar{y}^3} \tag{11}$$

Eq. 11 is the most important equation in designing lateral spillways in flumes of uniform rectangular cross-sections. (Analogous equations for other uniform canal shapes may be derived in the same way). For an established flume width b, velocity coefficient C_v, and energy head H_s at the flood stage, the rate of flow as a function of the depth can be computed. Plotting this data graphically in a y,Q-coordinate system yields the (y,Q)-curve.

As an example, the action of a lateral spillway for a 10-ft wide concrete flume with a slope of 0.00126 and with a roughness coefficient n (Manning) equal to 0.015 is investigated (Fig. 3a). The normal conveyance depth, y_n, is

6.0 ft for Q_n equal to 500 cu ft sec. At first, the velocity coefficient, C_v. is assumed to be equal to one. The maximum permissible depth, y_f, in the flume is 7.00 ft at 611 cu ft sec discharge and at an energy head of 8.19 ft. The elevation of the spillway crest, z, is assumed to be 3 in above the normal water level to prevent the water from splashing over it at normal flow. The intersection point of the crest line with the (y,Q)-curve in Fig. 3b represents the maximum theoretical diversion rate of 697 cu ft sec which the spillway may control at 0.75 ft maximum spillway head. However, the length of such spillway, when the water surface is level with the spillway crest at its beginning would be very long and the spillway itself inefficient. The reason for that is that the rise of the water level would not be initiated by the discharge action of the spillway, but only by the friction effect in its conduit. To start up the rise of the water level in the spillway section, a certain minimum head on the crest at its beginning is required.

As indicated by the graph in Fig. 2b, the velocity coefficient, C_v, in a spillway section may assume values up to 1.33. Therefore, neglecting the effect of the uneven velocity distribution in the spillway section may result in inadequate performance of the spillway. At first approximation, the magnitude of C_v may be determined using the depths obtained from (y,Q)-curve with uniform velocity distribution. Using the maximum crest head, $h_2 = \bar{y}_2 - z$, corresponding to the maximum conveyance flow rate in the canal, and the assumed minimum head, h_1, the average crest head, h_m, is determined. The approximate value of C_v obtained as a function of $\frac{h_m}{z + h_m}$ from the graph of Fig. 2b is somewhat smaller than the final C_v-value. The final C_v can be obtained repeating the same procedure with the corrected maximum depth. To be consistent in using the velocity coefficients, the normal water levels in spillway sections are computed with a C_v-value equal to 1.05 in all spillway examples in this paper.

In Fig. 4, the upper branch of the same (y,Q)-curve as in Fig. 3b is plotted in a larger scale as curve (1). The section of a heavy line below the thinner (y,Q)-curve represents the corrected water surface profile on the spillway crest with C_v equal to 1.17. As indicated in the Fig. 4, the maximum diversion flow rate at the flood water stage in the 10-feet-wide spillway section is only 634 cu ft sec at a minimum depth of 6.42 ft (0.25 ft above the spillway crest). The maximum depth at the end of the spillway for the maximum permissible flume flow of 611 cu ft sec at the flood stage is 6.67 ft. The maximum head on the spillway is 6.67 - 6.17 = 0.50 ft.

As reported in paragraph A-3d, the approach velocity upstream from the spillway should not exceed 0.75 times the critical velocity, i.e. the theoretical depth (at $C_v = 1.0$) at the beginning of the spillway should not be less than 1 1/3 times the critical depth. Otherwise, the flow may pass into the supercritical stage in spite of a subcritical bottom slope. As indicated by the intersection of (1 1/3 y_{cr})-curve with the (y,Q)-curve, it is quite close to the maximum diversion rate but still on the safe side.

The required length of the side spillway crest, L, is obtained by the summation of the discharges over the sections of the crest,

$$L = \frac{1}{C} \sum \frac{\Delta Q}{h_m^{3/2}} \tag{12}$$

The conjucated values $Q = Q_1 - Q_2$ and $h_m = \frac{h_1 - h_2}{2}$ in the crest sections can be easily computed by using Eq. 11 or scaled from the $(\bar{y},Q)$-curve in the range of the spillway.

. Efficiency of Lateral Spillway

The best criterion of the efficiency of a side spillway layout is the maximum rise of the headwater obtainable during the flood stage without encroachment on the minimum freebord of the canal.

In general, at a certain gate or orifice opening, the drop of the water level in an intake structure is about proportional to the square of the rate of the inflow. That means, the greater the spilling capacity of a lateral spillway, the higher is the possible rise of the headwater level during flood.

Using the discharge capacity of the lateral spillway as a criterion of its efficiency, the performance of the simple spillway layout, described in the preceding chapter, is very poor. As indicated by the $(\bar{y},Q)$-curve (1) in Fig. 4, the gradient of the depth variation in front of this spillway is rather steep. Besides that, the depth at the end of the spillway section, $\bar{y}_2$, is only 0.25 ft greater than the depth, $\bar{y}_1$, at its beginning. Because of this small change of depth and of the sharp downward curved section of $(\bar{y},Q)$-curve in the range of this spillway, the maximum diversion rate cannot exceed much the permissible conveyance flow of the conduit.

To improve this condition, the velocities in the spillway section are reduced:

1. By widening the spillway section to 15 ft or
2. By deepening it by 3 ft.

In this way, the range of the spillways will be shifted into the flatter sections of the (y,Q)-curves as shown on curves (2) and (3) in Fig. 4. The energy heads, H_s, during the flood flow are increased from 8.19 ft to 8.25 ft for the widened and to 11.26 ft for the deepened sections. As marked in Fig. 4 and in Table 1, the safe maximum inflow is increased from 634 cu ft sec in original 10 ft wide spillway section to 799 and to 805 cu ft sec for the widened and for the deepened spillway sections, respectively. However, the maximum spillway head, h_2, is increased from 0.50 ft only to 0.74 and to 0.79 ft respectively. Therefore, the lengths of the lateral spillways would be longer: 90 ft for the 15-ft-wide and 184 for the 3-ft-deepened sections.

Improvement of Performance of Lateral Spillways

As shown, widening or deepening of the spillway section would increase the spilling capacity of the lateral spillway. However, the spillways would then be very long because of the small heads on their crests. In order to reduce the length of the spillway, an additional orifice section may be placed between the spillway and the flume as shown in Fig. 5. This arrangement, using the same flow conditions in the flume as before, increases the permanent head loss during the normal flow, but cuts down the length of the spillway as indicated in Fig. 6 and in Table 1. Besides that, the length of the spillway can be shortened when the minimum head at the beginning of the crest is increased from the minimum 3 as used in all described layouts. However, the maximum diversion rate and therewith the possible headwater rise would then be reduced, too.

To demonstrate the efficiency of the orifice spillway, 3 alternate layouts are investigated:

1) An orifice of 53 sq ft net area is placed between the flume and spillway sections, both 10 ft wide. This orifice increases the energy head in the spillway section to 9.0 ft at the maximum flood flow when the water level in the flume is permitted to rise one foot;
2) An orifice of 43.4 sq ft net area is used to raise the energy head to 10. ft. The flume width is 10 ft as in (1); and
3) The section of the spillway is widened to 15 ft. The flood energy head and the net orifice area are the same as in (2).

The water level drop, $\bar{y}_o - y_f$, during the flood stage behind the orifice can be computed by the following equation:

$$\bar{y}_o - \bar{y}_f = \frac{(Q/C_o A_o)^2 - C_v V_o^2}{2g} \tag{13}$$

modified from the standard orifice equation.[16] In Eq. 13, $C_o A_o$ represents the net orifice area, assumed to be constant for normal as well as for flood flow; $\bar{y}_o$ and y_f are the respective water depths in front and behind the orifice and V_o is the mean approach velocity.

In comparison to the permissible flood diversion of 634 cu ft sec in a simple spillway layout of 10 ft width and of 8.19 ft energy head, the diversion rate at the flood stage, when orifices are used, is increased to 710, 825, and to 1155 cu ft sec, respectively, as shown in Fig. 6 and in Table 1. The most important advantage of an orifice spillway is the increased head on its crest. The maximum head at the end of the spillway is increased from 0.50 ft in the original 10-ft-wide spillway section to 1.21, 1.63, and 1.70 ft in the orifice spillways (1), (2), and (3), respectively. As a result, the respective lengths of the lateral spillways are 55, 98, and 334 ft.

As demonstrated by the 15-ft-wide orifice lateral spillway, the flood diversion rate could be increased considerably when the spillway section is enlarged. However, it is not advisable to use this extreme diversion rate because of too long and partly inefficient spillway crest. It is expedient to increase the minimum head on the crest and compensate the difference in the headwater rise by reduction of the net orifice area behind the spillway. In this way, the necessary crest length can be reduced.

As is recognizable from Fig. 6, the surface depression in the front of the spillway crests of the orifice spillways, illustrated by the heavy lined curves is much more distinct than in the simple spillway layouts. However, this is only accidentally so, because of very unfavorable head-depth ratios in front of the spillway crests which effect the most non-uniform velocity distributions with C_v-values 1.3 and over (Fig. 2b).

The stability of the flow in the orifice side spillways is good as shown by (1 1/3 y_c)-curves in Fig. 6, which intersect the (y,Q)-curves at Q-values we above the maximum diversion rates.

A disadvantage to the lateral spillways with an additional orifice is the increased permanent loss at normal flow.

16. "Handbook of Hydraulics" by H. W. King, 1955, p. 3-10.

. Application to Design Problem

In actual design, the headwater rise, ΔH, in a reservoir during the flood tage is determined by the characteristics of the drainage and storage areas, nd by the type and capacity of the discharge structures. Likewise, the free-ord of the canal is established with concern of the economics.

As shown in previous chapters, a higher headwater rise requires a higher pilling capacity of the lateral spillway. Besides that, economics of the tructure and the value of the permanent head loss in intake and in inserted rifice have to be considered. Further, the flow in the intake and in the later-l spillway sections should not pass into the supercritical stage.

In preliminary studies, it is useful to employ a simplified equation for es-ablishing the required net area of the intake gate opening, C_iA_i;

$$C_iA_i = \sqrt{\frac{Q_F^2 - Q_N^2}{2g(\Delta H - \Delta \bar{y}_1)}} \quad , \tag{14}$$

n which the constant orifice coefficient, C_i, is assumed to take care of all osses in the intake at any flow rates. Q_F and $\Delta \bar{y}_1 = \bar{y}_1 - y_n$, are conjugated alues established by Eq. 11.

As shown by Eq. 14, it is theoretically possible for any rate of floodwater ntake, Q_F, and headwater rise, ΔH, to establish a net gate opening. How-ver, there are physical and economical conditions which limit the size of e gate opening:

1) At certain Q_F-values, C_iA_i cannot be reduced beyond a minimum area, otherwise the flow behind the gate may pass into the supercritical stage,
2) At smaller gate openings, the permanent head loss during the normal flow may be prohibitively high; and
3) As demonstrated by (y,Q)-curves in Fig. 4, the required length of the spillway crest may be too long to be economical.

Further, Eq. 14 indicates that an increase of the water level rise, $\Delta \bar{y}_1$, at e beginning of the spillway helps to reduce the required maximum diversion ow at the same net gate opening. This way, the crest length of the spillway an be shortened.

Considering the foregoing statements, it is obvious that trials are neces-ary in designing economical and safe-acting lateral spillways. Being famili-r with the characteristics of the performances of different lateral spillway youts, explained in this paper, the designer will be able to cut down the num-er of trials.

CONCLUSIONS

In general, the lateral spillway itself is not capable of destroying any of e excess energy head upstream from it. The purpose of a side spillway in otecting an open canal against overflooding is merely to spill off the sur-us of the flow. The excessive energy head has to be absorbed upstream om the lateral spillway by the friction, intake and gate losses, or under rcumstances even by a controlled hydraulic jump. In a special case, when e spillway is located in front of the power intake to discharge the full or rt flow at the shut-down operation of the turbines, no energy head is

destroyed. On the contrary, the necessary energy head, to raise the water level, is built up by reduced friction loss in the backwater.

In respect to the computation and improvement of the lateral spillway, placed behind the diversion intake, the following conclusions can be drawn:

1) The performance of a lateral spillway can be computed using the Bernouilli and continuity equations. Because of the non-uniform velocity distribution in the lateral spillway section, the velocity coefficient, C_v, has to be considered in computations.
2) In uniform sections of the lateral spillway, the variation of the water level profile and the maximum spilling capacity, characteristic for each spillway layout, can be computed by Eq. 11.
3) It is useful to plot the (y, Q)-curves in designing lateral spillways. The relative location of the maximum diversion flow in the curve gives advance information about the spilling capacity and about the safety against the possible change of flow stage in the spillway section.
4) As demonstrated by the performances of different spillway layouts, the simple spillway with comparatively high velocities has a small spilling capacity. Consequently, the possible headwater rise is very limited.
5) Enlargement of the spillway cross-section helps to increase the spilling capacity and therewith the capacity to raise the flood level of the headwater. However, the spillway would be considerably long because of the small head on its crest.
6) An improvement can be obtained when an additional orifice is placed between the spillway and the canal. Although, the permanent loss at normal flow conditions is increased by the orifice loss, the length of the spillway is shortened considerably because of the higher head on its crest.

APPENDIX. NOTATION.

The following symbols, adopted for use in the paper, conform essentially with Letter Symbols for Hydraulics (ASA Z10.2 - 1942).

A_2 = area of the wetted cross-section at the end of a section, in sq. ft;

b = width of the section, in ft;

C = coefficient of spillway discharge;

C_y = correction coefficient of Bernouilli equation, by Schmidt;

C_v = coefficient of velocity distribution;

C_oA_o = net orifice area, in sq ft;

C_iA_i = net area of the opening of the intake gate, in sq ft;

g = acceleration of gravity, in ft per sec per sec;

h_1 = head on the crest at the beginning of a section, in ft;

h_2 = head on the crest at the end of a section, in ft;

h_m = average head on a spillway section, in ft;

H_F = energy head of the headwater at flood flow, in ft;

H_N = energy head of the headwater at normal flow, in ft;

H_s = energy head in the spillway section at flood flow, in ft;

ΔH = rise of the headwater at flood flow, in ft;

L = length of spillway crest, in ft;

M_1 = momentum in the upstream end of a section, in sec pound;

M_2 = momentum in the downstream end of a section, in sec pound;

ΔM_1 = momentum difference between the section ends, in sec pound;

ΔM = total momentum change in a spillway section, in sec pound;

n = coefficient of roughness in the Manning formula;

Q = rate of flow, in general, in cu ft per sec;

Q_1 = rate of flow in the upstream end of a section, in cu ft per sec;

Q_2 = rate of flow in the downstream end of a sect., in cu ft per sec;

Q_N = rate of diversion at normal flow, in cu ft per sec;

Q_F = rate of diversion at maximum flood flow, in cu ft per sec;

ΔQ = discharge across a section of the spillway crest, $\Delta Q = Q_1 - Q_2$;

S_f = slope of the energy head due to the friction loss;

S_{cr} = critical slope of the canal;

S = slope of the canal bottom;

V_o = mean approach velocity in front of an orifice, in ft per sec;

V_1 = mean velocity in the upstream end of a section, in ft per sec;

V_2 = mean velocity in the downstream end of a sect., in ft per sec;

ΔV = change of mean velocity in a section, $V = V_1 - V_2$, in ft per sec;

w = unit weight of water, in pounds per cu ft;

Δx = length of a section of the spillway, in ft;

y_n = normal conveyance depth of flume, with $C_v = 1$, in ft;

y_f = flood conveyance depth of flume, with $C_v = 1$, in ft;

y_{cr} = critical depth of flow, in ft;

Δy = variation of water level, in ft;

$\bar{y}$ = depth of water in the spillway section, with $C_v > 1$, in ft;

$\bar{y}_o$ = depth of water in front of an orifice, with $C_v > 1$, in ft;

$\bar{y}_1$ = depth of water at the beginning of the spillway, $C_v > 1$, in ft;

$\bar{y}_2$ = depth of water at the end of the spillway, with $C_v > 1$, in ft;

$\Delta \bar{y}_1$ = rise of flood water level at the beginning of spillway, $C_v > 1$, in ft; and

= elevation of the crest above the bottom, in ft.

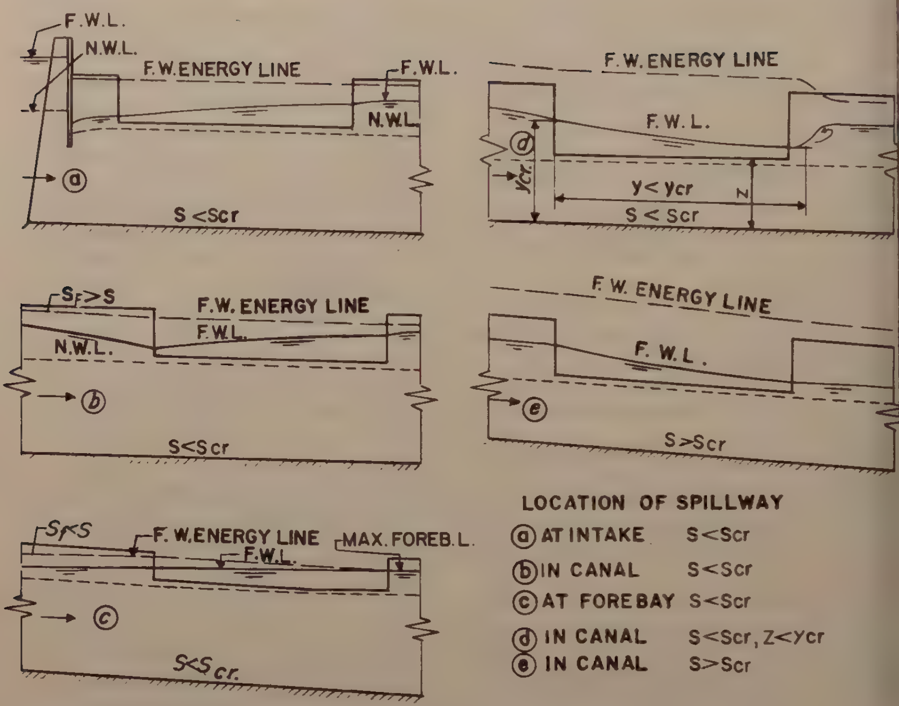

FIG.1–TYPICAL SURFACE PROFILES OF LATERAL SPILLWAY FLOW

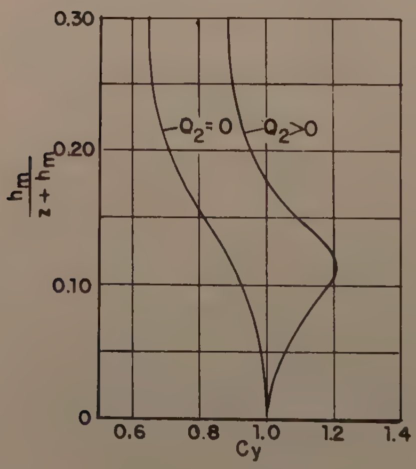

FIG. 2a–Cy VALUES

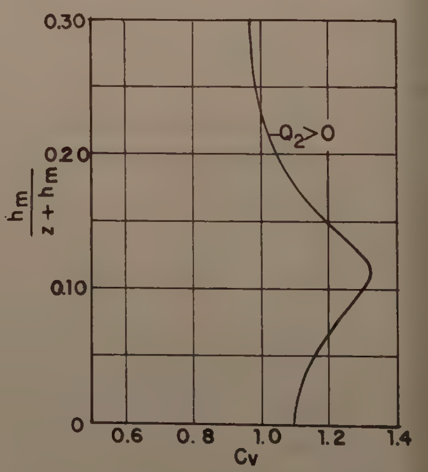

FIG. 2b–VELOCITY COEFF. Cv

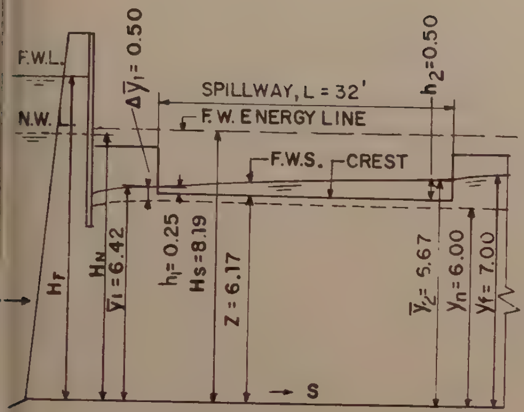

FIG. 3a – SIMPLE LATERAL SPILLWAY

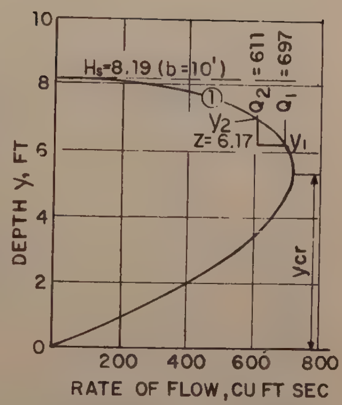

FIG. 3b – (y,Q) – CURVE

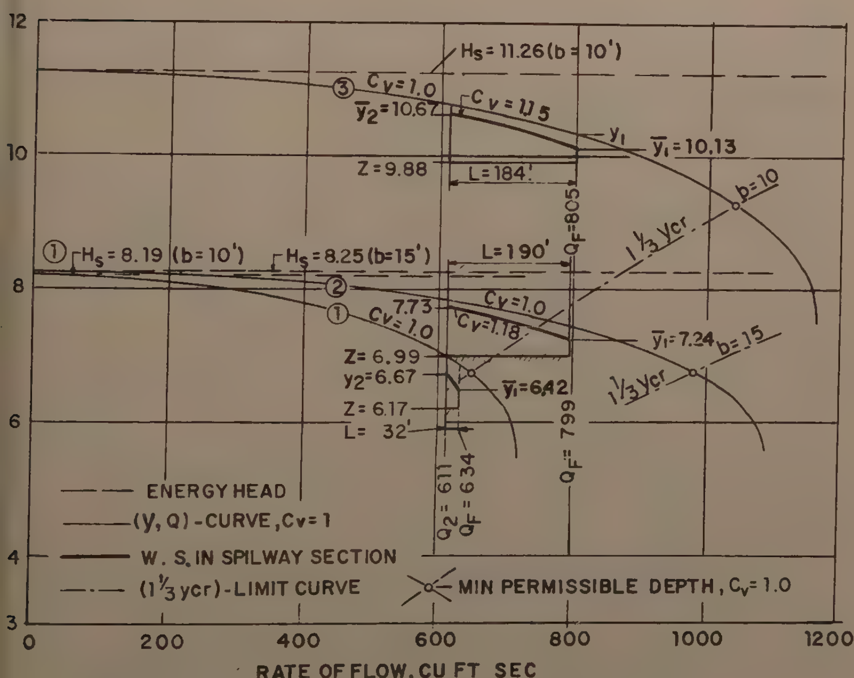

FIG. 4 – DESIGN GRAPHS OF SIMPLE LATERAL SPILLWAYS

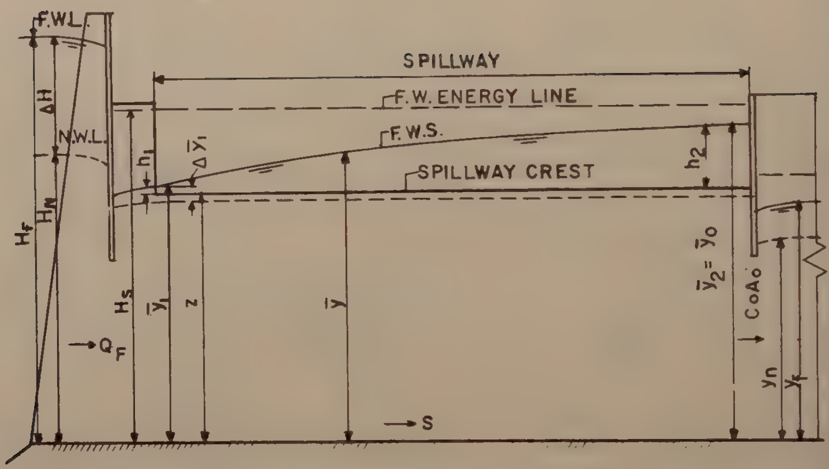

FIG. 5 - LATERAL SPILLWAY WITH ORIFICE

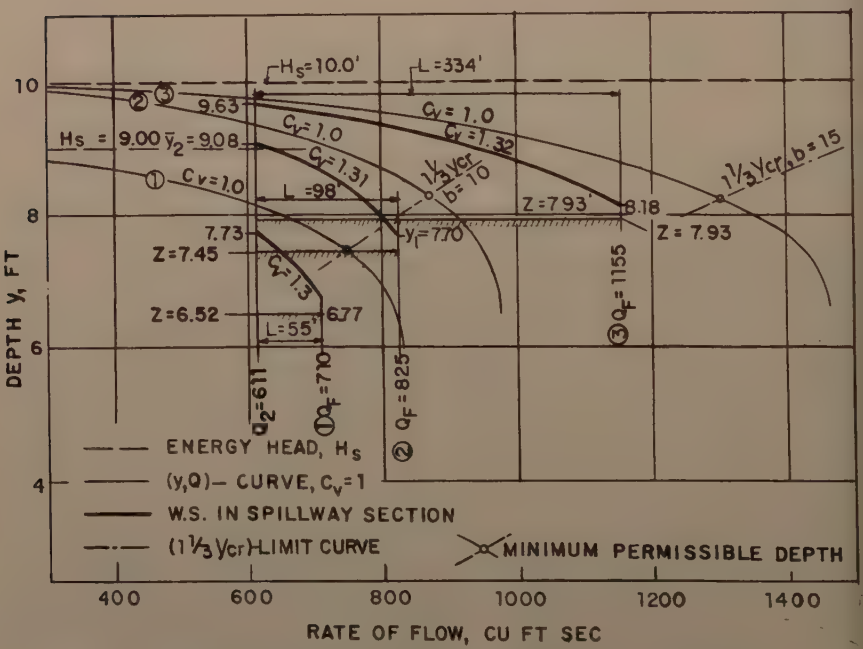

FIG. 6 - DESIGN GRAPHS OF LATERAL SPILLWAYS WITH ORIFICE

TABLE 1.- Performance Data of Sample Spillways

	Item	Simple Spillways						Orifice Spillways					
		(1) b = 10ft		(2) b = 15ft		(3) b = 10ft		(1) b = 10ft C_oA_o=53sqft		(2) b = 10ft C_oA_o= 43.4		(3) b =15ft C_oA_o=43.4	
		Norm.	Flood	Norm.	Flood	Norm.	Flood	Norm.	Flood	Norm.	Flood	Norm.	Flood
1	Rate of Flow in Conduit	500	611	500	611	500	611	500	611	500	611	500	611
2	Depth at Spillw.End } C_v=1.0	6.00	7.00	6.76	7.83	9.65	10.76	6.33	8.12	7.25	9.34	7.70	9.73
3	Velocity " " } C_v=1.0	8.33	8.73	4.93	5.20	5.18	5.68	7.90	7.53	6.90	6.54	4.33	4.19
4	Vel.Head " " } C_v=1.0	1.08	1.19	0.38	0.42	0.42	0.50	0.97	0.88	0.74	0.66	0.29	0.27
5	Energy Head in Sp.w.(2)+(4)	7.08	8.19	7.14	8.25	10.07	11.26	7.30	9.00	7.99	10.00	7.99	10.00
6	Vel.Coeff.C_v from Fig. 2b	1.05	1.17	1.05	1.18	1.05	1.15	1.05	1.30	1.05	1.31	1.05	1.32
7	Corr.Depth at Spillw.End,$\bar{y}_2$	5.92	6.67	6.74	7.73	9.63	10.67	6.27	7.73	7.20	9.08	7.68	9.63
8	" Vel. Head " ",(2)-(7)	1.16	1.52	0.40	0.52	0.44	0.59	1.03	1.27	0.79	0.92	0.31	0.37
9	Height of Crest, z=(7)+0.25	6.17		6.99		9.88		6.52		7.45		7.93	
10	Depth at Sp.w.Beg.$\bar{y}_1$=z+0.25		6.42		7.24		10.13		6.77		7.70		8.18
11	Flow Diversion, Eq.11	500	634	500	799	500	805	500	710	500	825	500	1155
12	Spillw. Capacity, (11)-(1)		23		188		194		99		214		544
13	Crest Head { Beginn.(10)-(9)		0.25		0.25		0.25		0.25		0.25		0.25
14	Crest Head { End, (7)-(9)		0.50		0.74		0.79		1.21		1.63		1.70
15	Length of Crest, Eq.12	32		190		184		55		98		334	

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

THE PROBLEM OF RESERVOIR CAPACITY FOR LONG-TERM STORAGE

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(Proc. Paper 1082)

SYNOPSIS

Until very recently there was no theoretical basis for the determination of the capacity needed in a long-term reservoir to guarantee a given draft. Lately, a solution of the problem has been given by Hurst on the assumption that hydrological occurrences may be treated as random events the sequence of which is free of any periodic trend. Hurst has shown that, subject to that condition, the main factor by which the capacity is governed is the standard deviation of a single observation.

Considering that in hydrological phenomena there is usually some tendency towards grouping of high or low years, the authors have approached the problem from a different angle. In the new treatment, deviations in the arithmetic mean for groups of observations are utilised instead of the deviation of one observation. In this way, the influence of any peculiarity of sequence in the phenomenon under consideration will obviously be covered.

INTRODUCTION

Ideal Form of the Problem

In the design of over-year storage schemes for irrigation, power generation or town water-supply an important consideration is that of reservoir capacity. In its simplest form, the problem consists in the determination of the capacity required to secure a constant draft equal to the mean supply. The fact, however, that it is usually impossible to make an accurate forecast of the supply for a long time in advance renders the notion of a draft exactly equal to the mean supply impracticable. Nevertheless, that notion constitutes the logical starting point for a theoretical study of the problem. A general idea of the fundamental principles involved will be gained from the following example:

Note: Discussion open until March 1, 1957. Paper 1082 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 82, No. HY 5, October, 1956.
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Suppose we are given a set of observations of variable magnitude representing the annual discharge of a river at a certain site, and it is required to find the capacity needed in a balancing reservoir with the aid of which we can secure a constant annual draft equal to the mean of these observations.

It is easy to see that the capacity required equals the range between the maximum and the minimum values of the cumulated departure of individual observations from the mean. This is illustrated in the following table:

Table I

No.	Obs.	Departure +	Departure −	Cum. Dep.	Remarks
1	36	16		16	
2	28	8		24	Max.
3	12		8	16	
4	16		4	12	
5	12		8	4	
6	8		12	− 8	
7	16		4	− 12	Min.
8	26	6		− 6	
9	32	12		6	
10	14		6	0	
Mean	20				

In this case, the storage needed will be the range between 24 and -12, which is 36.

Another way in which this result may be arrived at is to tabulate the cumulated draft against the cumulated supply. The difference between the two at the end of each year will be the reservoir content at that instant. This is shown in the next table.

A peculiarity in Table II is that at certain times the reservoir contents become negative. A constant draft equal to the mean cannot, therefore, be obtained throughout the whole of the working period unless we borrow a volume at least equal to the maximum negative storage. Supposing a volume of 12 to be borrowed at the start, the adjusted reservoir contents will be as shown in the last column. Obviously, the residue in the reservoir at the end of the operation must correspond to the borrowed volume since the total outflow equals the total input.

Graphical Representation

The graphic plot of either the cumulated supply or the cumulated draft is

Table II

No.	Obs.	Cum. Supply	Cum. Draft	Res. Con.	Adjusted Res. Cont.
		0	0	0	I2
I	36	36	20	I6	28
2	28	64	40	24	36 Max.
3	I2	76	60	I6	28
4	I6	92	80	I2	24
5	I2	I04	I00	4	I6
6	8	II2	I20	- 8	4
7	I6	I28	I40	-I2	0 Min.
8	26	I54	I60	- 6	6
9	32	I86	I80	6	I8
I0	I4	200	200	0	I2
Mean	20				

called a "Mass-Curve." The mass-curves for the case assumed above are shown in Fig. 1. The curve Oabc represents the cumulated supply. The straight line Oc represents the cumulated draft from an empty reservoir, while line O'c' represents the cumulated draft with an initial storage of 12. The following points are useful to note:

1) The slope of a mass-curve at any point represents the rate of flow.
2) For a constant rate of flow the mass-curve is a straight line.
3) The reservoir content at any instant is given by the respective intercept between the mass-input and the mass-outflow curves.
4) The range is given by the intercept between two lines drawn parallel to the output line through the highest and lowest points on the input curve in relation to that line.
5) The mean up to any point on a mass-curve is given by the slope of a straight line drawn from the origin to that point. Similarly, the mean for any intermediate period is given by the slope of a straight line joining the initial and the terminal points on the mass-curve.

The intermediate period determining the range (from a to b in Fig. 1) will here be referred to as the "Critical Period." In the above example, the critical period is a period of deficit. According to its definition, the range may also correspond to a period of excess. That depends on the order in which the points of maximum and minimum cumulated departure are encountered.

Denoting the length of the critical period by N_c, the respective mean by M_c, the overall mean by M_o and the range by R, it is clear that:

$$R = N_c \left| M_o - M_c \right| \tag{1}$$

Variation of Range with Number

If we take a long series of observations of non-periodic character and compute the ranges for sets of different length, we shall generally find that the range tends to increase progressively with the number of observations taken. Thus, under the conditions assumed above, the capacity needed in a long-term reservoir would be indeterminate unless the length of the working period were fixed. A theoretical study of that problem has recently been made by Dr. H. E. Hurst in connection with storage schemes in the Nile Basin.(1) He has succeeded in establishing a relation between the mean range (R), the number of observations (N) and the standard deviation (σ), for random events of normal frequency distribution thus:

$$R = 1.25\, \sigma \sqrt{N} \tag{2}$$

Noting that

$$\sigma = \sqrt{\frac{\Sigma v^2}{N}}$$

where v = deviation of a single observation from the absolute mean, the above expression is equivalent to:

$$R = 1.25 \sqrt{\Sigma v^2} \tag{3}$$

showing that the range increases in the same manner as the accumulated error on a line of levelling.

Finding that the range for natural phenomena did not, in general, conform to that law, Hurst assumed a similar relation of the form:

$$R = a\, \sigma\, N^k \tag{4}$$

where the coefficient a and the index k had to be determined empirically.

If the best values of a and k were extracted for each phenomenon independently and without, in any way, predisposing the result to be expected, it would be found that the range of variation therein from one phenomenon to another is somewhat large. The result of an analysis carried out by the authors on rainfall observations at seven stations, each with a record of not less than 100 years duration, is given in Table III.

To gain some idea of the influence of this variation on the range, when relation (4) is applied, let us take N = 100 as a standard for purposes of comparison. For Madras $R/\sigma = 9.5$. For Boston, $R/\sigma = 25.9$.

In dealing with this problem, however, Hurst supposed that the coefficient a and the index k in (4) should be such as to satisfy the condition $R/\sigma = 1$ when N = 2. On this supposition (4) was reduced to the form:

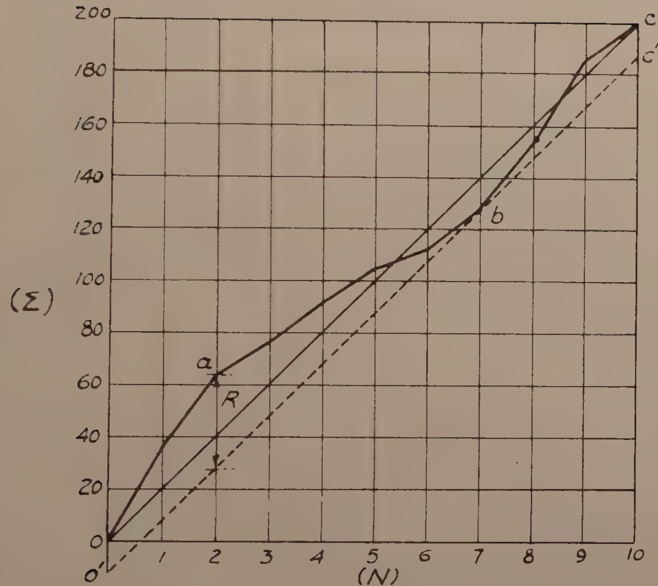


Fig. I - MASS-CURVES

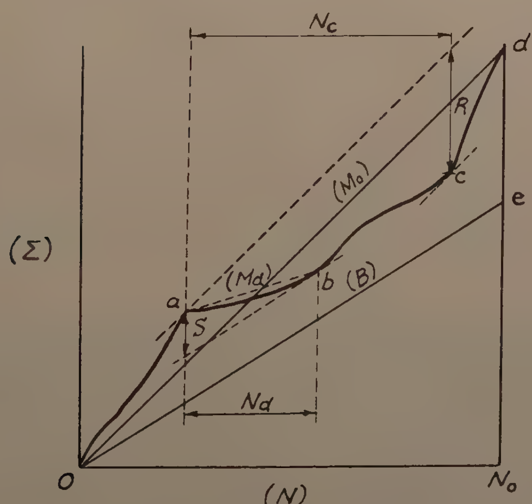


Fig. 2 - MAXIMUM DEFICIT

Table III

Station	a	k
Copenhagen (Denmark)	0.35	0.79
Charleston (U.S.A.)	0.34	0.94
Boston (U.S.A.)	0.13	1.15
Philadelphia (U.S.A.)	0.13	1.15
New York (U.S.A.)	0.77	0.74
Madras (India)	1.82	0.36
Edinburgh (Scotland)	1.82	0.43

$$\frac{R}{\sigma} = \left(\frac{N}{2}\right)^k \qquad (5)$$

Thus the coefficient (a) was tied up with the index (k). By logarithmic plotting of R/σ against N, in which all lines were made to pass through the point (0, log 2), it was found that variation in k was confined within a reasonable range for all sorts of natural phenomena such as river levels and discharges, rainfall, temperature and pressure, annual growth of tree rings, mud deposits in lakes, etc. For that index an average value of 0.72 was adopted. It followed that the coefficient (a) should have the value 0.61, whence relation (4) acquire the final form:

$$R = 0.61 \, \sigma \, N^{0.72} \qquad (6)$$

In the authors' opinion, that procedure would be justifiable only if the relation were in the nature of an exact law. Considering that the condition R/σ = when N = 2 is far from being satisfied in relation (2), for which there is a theoretical foundation, it would appear that the consistency apparently gained through that extrapolation is the product of the procedure itself and not of any uniformity in the character of the various phenomena examined.

Case of a Draft Less than the Mean

The question of the storage needed to guarantee a draft less than the mean which is the more important from the practical point of view, was considered by Hurst on the supposition of the existence of a relationship between that storage and the normal range.(1) His treatment of this subject may be summed up as follows:

Let N_o = overall number, M_o = overall mean, B = draft (less than M_o) and S = storage required to guarantee that draft. The difference between B and M_o was expressed as a fraction of the standard deviation, and S was taken as the greatest accumulated deficit, within the period N_o, with respect to the draft B.

The notion of the maximum deficit in relation to a given draft is demonstrated in Fig. 2. The range (for a draft = M_o) is given by points a and c. For the draft B, as represented by line Oe, the maximum deficit is given by points a and b. The corresponding storage S must obviously be smaller than R.

Through analysis of several sets of random and natural observations, the following empirical relations were arrived at:

For random events, $$\frac{S}{R} = 0.91 - 0.89\sqrt{\frac{M - B}{\sigma}} \qquad (7)$$

For natural events, $$\frac{S}{R} = 0.97 - 0.95\sqrt{\frac{M - B}{\sigma}} \qquad (8)$$

It was admitted that a great deal of research was devoted to finding a theoretical relation between S and R but without success.

Evidently, the first numerical term in either (7) or (8) should have been unity since S should have the same value as R when (M-B) = O. The discrepancy is due to the fact that S has to be taken as the maximum deficit only whereas R may correspond either to a deficit or to an excess.

It is to be noted, however, that once the notion of a draft exactly equal to the mean is relinquished, the situation undergoes a substantial change. With a draft equal to the mean, the range generally increases progressively as the number increases. This cannot always be the case with a reduced draft. In fact, if the draft were reduced to the minimum value of a single observation, S would be zero whatever the number. Thus there are no grounds for the supposition of a fixed relation between S and R.

Practical Considerations

In Art. 1 above it has been pointed out that the notion of a draft exactly equal to the mean is impracticable in view of the fact that that mean could not be accurately known in advance. Another reason is that, even if the mean were known, some initial storage would, as a rule, be needed to eliminate negative reservoir contents and the amount of that storage could not possibly be determined beforehand. Thus, regulation would almost certainly have to deviate from the ideal program visualised and, once that deviation took place, the theoretical structure built up on that notion would tumble down.

In addition to these considerations, there are two points which have an important bearing on the storage problem from the practical side. The first is that the capital outlay for a long-term storage scheme is usually high and the capacity needed should be computed on the most specific lines possible. In other words, as much regard as possible should be paid to the peculiarities of the case in hand. Thus it would be inadvisable to rely on formulae derived through taking averages for a large number of phenomena of different character.

The second point is that a small reduction in the draft below the mean is usually found to lead to a relatively huge reduction in the capacity needed. Thus we should always allow for some discount in the draft below the estimated mean, apart from the allowance for storage losses, even if the whole of the supply were needed for consumption. That discount would serve both as a factor of safety (in the absence of accurate knowledge of the future mean) and as a means of securing a great economy in cost with a little sacrifice in benefit.

With these considerations in mind, the authors have attacked the problem from a new angle, regard being paid in particular to the case of a draft less than the mean. It is to be borne in mind, however, that the problem is one that cannot be solved successfully by hard and fast rules and that, whatever the line of approach, the procedure cannot be freed entirely from the element

of conjecture. The most effective tools in the hand of the designer will be sound practical judgment and a thorough understanding of the fundamental principles involved.

Characteristics of Hydrological Phenomena

General Considerations

The principal phenomena with which we are concerned in the present study are rainfall and river flow. In subjecting any such phenomenon to systematic analysis, it is tacitly assumed that the occurrence repeats itself regularly and that we can rely on obtaining an average annual supply of a certain order for an unlimited number of years. Thus, we must exclude from our discussion such phenomena as occur only at long intervals or in widely varying degrees of intensity. It has also to be assumed that the occurrence under consideration, though variable from year to year, is not subject to progressive secular change.

The first question that presents itself here is how far hydrological phenomena may be treated as random events. One respect in which a difference is most likely is that of the frequency distribution. Most phenomena of that kind show skew distributions, positive deviations from the mean being generally higher and less frequent than negative deviations. This will be readily understood when it is noted that while there may be years of more than 100% excess above the mean, there cannot possibly be any year with more than 100% deficit.

Another distinctive feature in hydrological phenomena is that high and low years often occur in runs or long-range cycles. Although these cycles do not, as a rule, take place in symmetrical waves or at regular intervals they are likely to affect the storage question considerably. In the case of a purely random occurrence, it may be taken as an established fact that the normal range generally increases as the square root of the number of observations when that number is fairly large. The authors have applied Hurst's formula (2) to observations obtained by throwing five dice a thousand times, the sum obtained at each throw being recorded as one reading. Although deviations on one side or the other took place, the average range agreed very closely with the theoretical value. With natural phenomena, however, the average range has been found to increase with number in some cases at a slower rate and in others at a higher rate than indicated by either (2) or (6). This is due mainly to the peculiarities of sequence in such phenomena.

Many attempts have been made to prove the existence of regular periodicities in meteorological or hydrological phenomena but, so far, without success. An example is the theory put forward by Brooks that the levels of Lake Victoria fluctuate in unison with the number of sunspots, which is known to change more or less periodically. That theory has been questioned by Hurst and Phillips on several grounds.[2]

With the object of detecting periodicities in the flow of the River Nile, an analysis of the maximum flood levels on Roda Guage, of which the records extend back for 1300 years, was made by Jarvis and discussed by Hurst, Shumann and others.[3] No definite conclusions were arrived at. It is reasonable to suppose that some correlation must exist between hydrological phenomena and periodic cosmical phenomena. The multiplicity of factors involved, however, renders any inquiry in this respect futile.

A simple method of revealing cyclic trends in rainfall or river flow records is to draw curves representing 3-year means, 5-year means, 7-year means and so on, each mean value being plotted against the median year of the corresponding period. To obtain smooth mean-curves, the subordinate periods should not be taken consecutively, i.e. each beginning at the end of the preceding one, but should be shifted forward only one or two years at a time. Fig. 3 shows a plot for the rainfall at Boston (U.S.A.). For purposes of comparison, a similar plot for the 5-dice observations is also given.

Incidentally, it may be mentioned that if a definite long-range periodicity exist, the operation of taking long-period means progressively as suggested above will not obscure that cycle, provided no other cycles are present. This fact may be demonstrated as follows:

Suppose we have a function y = sin(nx) and we take means by small shifts for a subordinate period of length 2z. The difference between an individual observation and the coincident mean will be:

$$\sin(nx) - \frac{1}{2z}\int_{x-z}^{x+z} \sin(nx)\, dx = \sin(nx)\left[1 - \frac{\sin(nz)}{nz}\right] \quad (9)$$

Thus, the effect of taking means is simply a distortion of the y-scale without any alteration in the period length. If superimposed deviations are haphazard, their effects on the mean will cancel out.

The Mean

Before discussing the characteristics of the mean in natural phenomena, it may be useful to review these characteristics briefly as regards purely random events. The latter are generally subject to the following conditions:

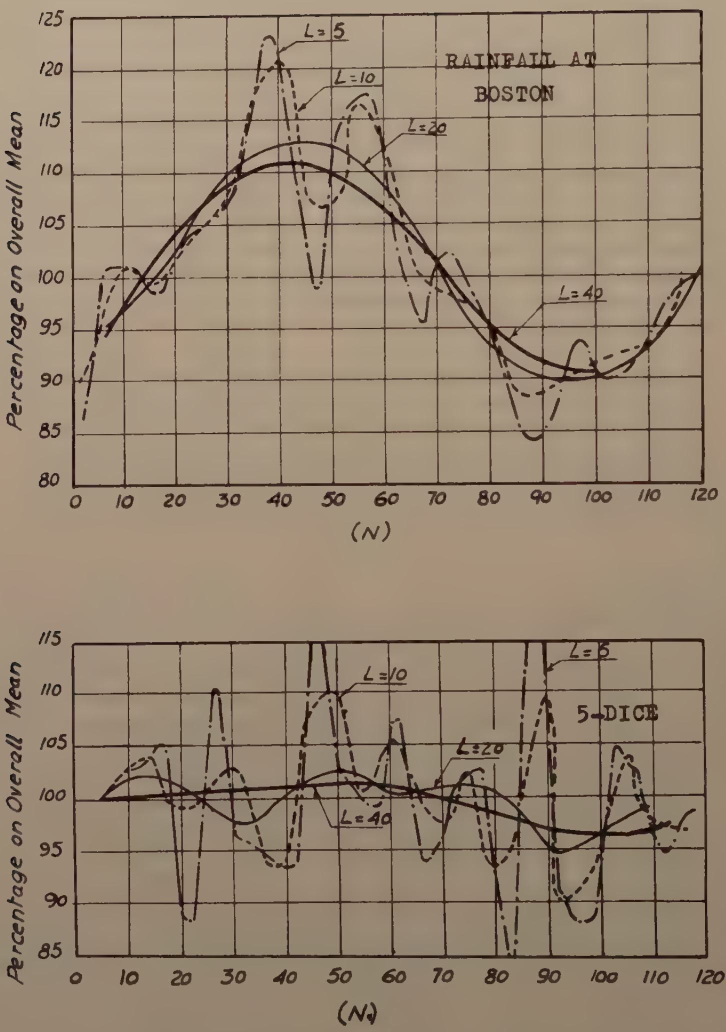
1) Positive and negative departures from the mean are equally likely.

2) There is usually a theoretical or absolute value of the mean. In the case of the 5-dice, for example, the maximum possible reading is 30 and the minimum is 5. The absolute mean, therefore, is 17.5.

3) The absolute mean is most likely to fall within the range of observed values, provided the number of observations is not very small (not less than, say, 10 or 12).

The significance of these conditions is perhaps most clearly brought out in the case of the repeated measurement of a certain dimension such, for example, as the distance between two points. If there is no systematic error in the process of measurement, we may expect that beyond a few trials the true length sought will lie within the range of the observed values and not very far from their arithmetic mean. In other words, it will be most improbable that all the observed values will be greater or less than the true value.

Natural phenomena generally differ from random events in all of these respects. The absolute mean is always indeterminate and individual values may lie above or below the overall mean adopted for long periods on end. The condition of equal probability of positive and negative deviations is also not satisfied in the majority of cases. Under these circumstances, there will be no guarantee that the mean for a given number of future years will not deviate sensibly from the mean derived from past observations. On this account, consideration of possible variations in the mean becomes a matter of paramount importance.



(L = Period Length or Number of Observations)

Fig. 3 - PROGRESSIVE VARIATION IN PERIOD-MEAN.

A study of fluctuations in the mean annual rainfall at a large number of stations situated in different parts of the globe was made long ago by Binnie. Binnie.(4) His main conclusions are:

1) To obtain a reliable value of the mean, the existing record should extend over a period of at least 35 years. This was supposed to yield a result correct to within 2% of the mean for any longer period.

2) The extreme positive deviation in a subordinate-period mean from the overall mean is generally greater than the extreme negative deviation.

3) The range of fluctuation in the mean is generally greater for stations of low rainfall than for those of heavy rainfall.

As regards the first point, it may be mentioned that in calculating the subordinate-period means consecutive periods only were taken. The observed maximum deviations were thereby greatly under-rated owing to the consequent reduction in the number of periods tested as the period length was increased. To draw the fullest possible information from the available records, overlapping periods should have been taken, as suggested in the preceding article.

The authors have made an analysis of rainfall records at ten stations, each of not less than 100 years duration, to find out the effect of increasing the number of shifts for a given period-length. The result is shown in Fig. 4. In the compilation of that figure, the average extreme deviations for the ten stations were taken. For the sake of uniformity, the subordinate periods were shifted five years at a time and the percentage deviation was referred to the mean of the first period (which would be the only one known when an attempt is being made to estimate the future mean). This analysis shows that much greater deviations than those found by Binnie are possible, even with periods exceeding 35 years.

As regards the second point, the cause of the distinction between positive and negative deviations has already been referred to in the preceding article.

The third conclusion indicates that heavy rainfall is equivalent to a combined supply from several sources between which there is no correlation. This may be demonstrated as follows:

Let there be two exactly similar sources, each having a mean M and a standard deviation σ. The ratio σ/M may be taken as a measure of relative deviation for one source. For the two sources together, the resultant mean = 2M and the resultant standard deviation $=\sqrt{2}\sigma$. Thus, the relative deviation for the combination will be 0.707 of the relative deviation for one source.

The question of the maximum deviation in the arithmetic mean, to which reference has been made before, is one of major importance in the present study. A clearer view of its significance will be gained if we discuss it first from the theoretical point of view.

For random events, it is known that the probable deviation of the arithmetic mean is:

$$\text{P.D.} = \pm \frac{0.6745}{\sqrt{N}}\sigma$$

This represents the deviation with respect to which the actual deviation is equally likely to be greater or smaller in magnitude. It is also known that the probability of a deviation greater than a certain multiple of the P.D.

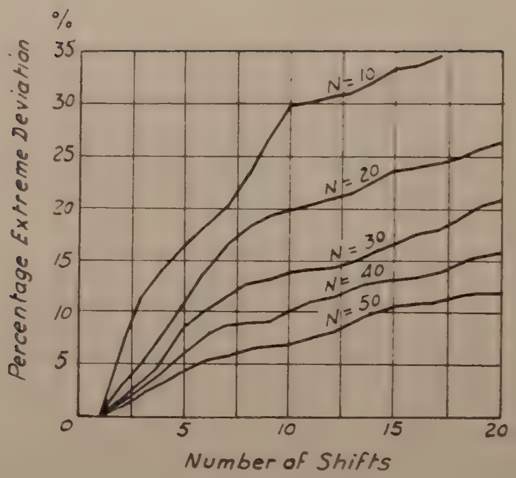


Fig. 4 - PERCENTAGE EXTREME DEVIATION IN SUBORDINATE-PERIOD MEANS

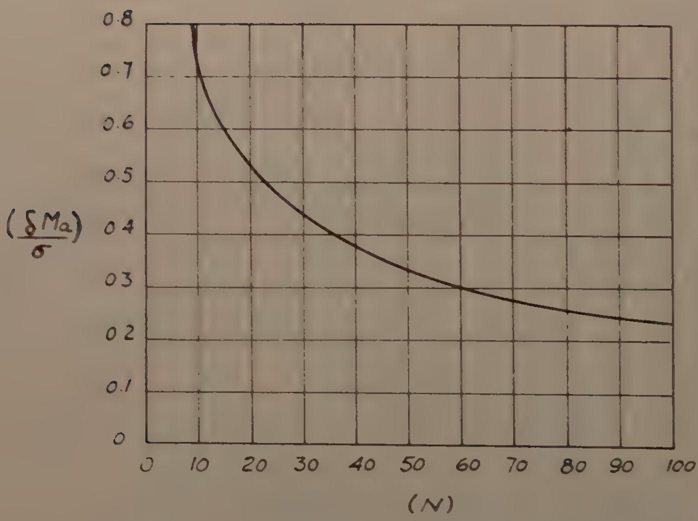


Fig. 5 - DEVIATION-CURVE FOR RANDOM EVENTS

decreases very rapidly as the proportion increases. Some values of that probability are given in the following list:(5)

Ratio to P.D.	Probability
1.00	1/2
2.44	1/10
3.06	1/25
3.45	1/50
3.82	1/100
4.85	1/1000
5.86	1/10000

It is quite logical to suppose that probability conditions with respect to the arithmetic mean (regarded as an observation in itself) must be similar to those relating to single observations, with the difference that the standard deviation for the former = $\sigma/\sqrt{N}$ instead of σ. Thus we can say that the probability of meeting with a deviation greater than, say, 3.45 times the P.D. in a set of observations of the mean for some fixed number of individual observations = 1/50. Or, to put it the other way, if we have 50 values of the mean for a given number N then, on the average, one of these will depart from the absolute mean by 3.45 times the P.D. or more, on the understanding that the P.D. is that corresponding to the number N.

Although the deviations represented in Fig. 4 are referred to the mean of the first period and not to the absolute mean, the plot shows clearly that natural phenomena are subject to the same general trends. It is also to be borne in mind, however, that variations in the mean for such phenomena are most likely to be influenced by peculiarities of sequence as well as by the standard deviation. On this account, it is not considered admissible to apply the above mentioned rules, without reservation, to any natural occurrence. Further reference to this question will be made later.

The Standard Deviation

Just as we cannot assign an absolute value to the mean for natural phenomena, so is the situation as regards the standard deviation. Observation shows that in such phenomena the range of fluctuation in the standard deviation is generally much wider than the range of fluctuation in the mean. For example, the means and standard deviations for the rainfall at Greenwich in three consecutive periods, each of 30 years duration (1841-1930), are: 614, 602, 629 and 120, 83, 108, respectively. In these figures, the highest value of the mean is 4.5% greater than the lowest, while the highest value of the standard deviation is 45% greater than the lowest. Another striking example is furnished by the annual mud deposits in Lake Sake (Crimea). For two consecutive periods, each of 100 years, the means and standard deviations are: 10.9, 15.8 and 3.9, 21.0, respectively. In this case, the higher value of the mean is 45% greater than the lower, while the higher value of the standard deviation is 440% greater than the lower. These figures are taken from tables attached to Hurst's paper already referred to.(1) According to Hurst, the average range of variation of the mean rainfall, for periods of 40 to 50 years, is 14%, while the corresponding range for the standard deviation is 23%.

The Range of Cumulated Departures

A study of the manner of variation of the range of cumulated departures with number will provide us with a guide as to how far natural phenomena may be regarded as truly haphazard. As Hurst has established theoretically, the mean range varies directly as the square root of the number, in random events of normal frequency distribution. The coefficient of proportionality depends on the standard deviation but, as far as the general behaviour of the range is concerned, it is immaterial on what that coefficient depends so long as it is a constant for the phenomenon considered.

From tests carried out by the authors on the direct relation between range and number for natural phenomena, it appears that the range generally increases with number more rapidly than for random events, the exponent of N in some cases being very near the value unity. Owing to the shortness of the records, however, no general conclusions can be drawn from these tests.

In this respect, the authors are inclined to the view that the relation between range and number is influenced not only by the standard deviation but also by the peculiarities of sequence which are observable in almost all natural occurrences. This will be readily understood when it is noted that for a perfectly periodic phenomenon the range has a limiting value which is never exceeded no matter how great the number.

Irregular cycles, on the other hand, may have a reverse effect on the range, i.e., may cause it to increase with number more rapidly than when the distribution of individual observations is absolutely random. On this account, it is believed that every phenomenon should be dealt with according to its own particular circumstances.

New Line of Approach to the Problem

Basis of Proposed Treatment

In the proposed treatment of the long-term storage problem attention is paid in particular to the case of a draft less than the mean, which is the only one of practical importance. For purposes of demonstration, however, we shall start with the ideal case of a draft exactly equal to the mean.

Going back to Eq. 1, it will be seen that the range is simply the outcome of deviation in the arithmetic mean for some intermediate period from the overall mean for a given number of observations. If the phenomenon is supposed to possess an absolute mean (M_a), the overall mean (M_o) will rarely coincide with M_a, despite the fact that the latter is the "most probable" value of the mean for any period. Hence we can say that the range is governed by three factors, namely:

1) The length (N_c) of the critical period.
2) The deviation of the critical-period mean (M_c) from M_a.
3) The deviation of M_o from M_a.

If δ is the relative deviation of any period mean with reference to the absolute mean, it follows from (1) that:

$$\frac{R}{M_a} = N_c \left| \delta_o - \delta_c \right| \tag{10}$$

For a given value of N_O, there are many possible combinations between N_c, δ_O and δ_c. Each of these factors will have a probable value of its own. It is obvious, however, that we cannot obtain the probable value of R/M_a directly by inserting the probable values of the above mentioned factors in (10).

There may be a means of elaborating that problem on theoretical considerations but it is not considered worth while to make that attempt because the probable range itself is of doubtful practical value. The adoption of the probable or the average range would mean that the chances of satisfying or not satisfying the prescribed conditions would be about the same. In other words, it is the maximum and not the mean range that we should endeavour to ascertain.

One way of dealing with that problem is to adopt a standard value for the ratio N_c/N_O and lay down some empirical rule for the choice of suitable values of δ_O and δ_c. This is the essence of the proposed method of tackling the problem. In the ideal case visualised, this procedure may appear somewhat arbitrary but the method is particularly suited to the case of a draft less than the mean, as will be shown presently.

In the application of that method, the essential natural characteristic of the phenomenon under consideration to be utilised is the maximum deviation of the arithmetic mean for any given number from the absolute mean. That characteristic has already been considered for random events in Art. 7. As regards hydrological phenomena, it will be assumed that we can construct a curve representing the maximum relative deviation that may be expected, within reason, to take place in the mean for a given number, with reference to a hypothetical absolute mean.

To make clear that notion, we shall first show how such a curve may be constructed for a random event.

In Art. 7, a scale has been given of the greatest deviations that are most likely to be met with in various numbers of observations of the arithmetic mean. Theoretically, there is no limit to the relative deviation that may be attained. In practice, however, the magnitudes of individual occurrences are usually confined between certain physical limits beyond which the mean cannot possibly pass. But the probability of attaining any of these limits with sets of more than 3 or 4 observations is practically nil since that could happen only if all the readings coincided with either the maximum or the minimum value.

Considering that for deviations in the arithmetic mean exceeding a few times the P.D. the probability becomes very small, an arbitrary limit may be set at some probability which is considered low enough. Supposing we take that limiting probability = 1/50, the corresponding value of the most likely maximum deviation may be taken = 3.5 times the P.D. On this basis we may write:

$$\text{Max.D.} = \pm \frac{2.36}{\sqrt{N}}\sigma \qquad (11)$$

$$\text{and} \left|\delta\right|_{\text{max.}} = \frac{2.36\,\sigma}{\sqrt{N}\,M_a} \qquad (12)$$

The plot of $|\delta|_{\text{max.}}$ against N will be called a "Deviation-Curve." From (12) it is clear that the relation between the two depends on the ratio σ/M_a which generally varies from one phenomenon to another. Fig. 5 shows a plot of the quantity $\delta M_a/\sigma$ against N.

The next question to consider is the relative length of the critical period. As already pointed out, the critical period may be either one of deficit, as shown at (a), Fig. 6, or one of excess as shown at (b). In the case shown at (c) in the same figure, either the excess period 0a or the deficit ac may be regarded as the critical period.

It would be natural to suppose that if deficits and excesses were equally likely, the most probable value of N_c would be $N_o/2$. This probability is most apparent in the last case. Analysis of the 5-dice observations, however, shows that the average value of the ratio N_c/N_o is somewhat less than 1/2 and that it tends to decrease slowly as N_o increase. For numbers between 20 and 200, that average has been found to conform closely to the relation:

$$N_{c/N_o} = 0.67\, N_o^{-0.I} \tag{13}$$

This result gains some weight from the fact that the same relation has been found to hold for natural as well as for random events. It is evident, however, that a highly refined treatment of that point would be superfluous. It is, therefore, proposed to adopt for that ratio a standard value of 1/2.

Finally we come to the question of what values of δ_o and δ_c we have to adopt in relation (10). It stands to reason that if we assign to δ_c its maximum value, as previously defined, we can assume $\delta_o = 0$ on the ground that positive and negative values of that deviation are equally likely. Strictly speaking, that argument is not quite correct because N_c is a fairly large fraction of N_o and any deviation in M_c from M_a must to some extent be reflected in M_o. In other words, the most probable value of δ_o in combination with the maximum of δ_c would be a small quantity of the same sign as δ_c. That, again, is a refinement for which there is no need. Relation (10) may now be reduced to the form:

$$\frac{R}{M_a} = N_c\left|\delta_c\right| \tag{14}$$

on the understanding that $N_c = N_o/2$ and δ_c is the maximum for the period N_c.

Substituting for N_c and δ_c is (14) their equivalents in terms of N_o and σ we get:

$$R = I.66\,\sigma\sqrt{N_o} \tag{15}$$

Comparing (15) with (2), we see that the maximum range according to the foregoing procedure is about 33% greater than the mean range. This result agrees well with actual tests carried out on the 5-dice observations.

The Deviation-Curve for Hydrological Phenomena

The deviation-curve for a natural phenomenon is obtainable only through direct analysis of the existing observations. It could be readily derived from such a plot as shown in Fig. 3. It is admitted that if the available record is short, the construction of that curve will be difficult. But, in that event, it will be equally difficult to make a reliable estimate of the standard deviation

which is the only alternative to employ. In any case, some judicious adjustments to the observed maxima of the deviations in the means will be necessary.

The maximum deviation for a given number of observations may generally be expressed by a relation of the form:

$$|\delta|_{max.} = \frac{c}{N^m} \tag{16}$$

where the best values of c and m have to be determined for each phenomenon independently. If the available record is short, then we may either use the theoretical relation for random phenomena (II) or adopt some relation derived for another record of long duration of which the general characteristics are believed to be similar to those of the phenomenon under consideration.

As there are only two unknowns in (16), the entire curve could be drawn when δ is known for any two values of N. It is preferable, however, to work out the largest possible number of values of δ and draw an envelope to the plots of these values. After that the values of c and m could be determined by selecting two suitable points on that envelope.

It is to be noted, however, that in hydrological phenomena the maximum positive deviations are likely to be relatively greater than negative deviations (see Art. 7). As far as the range (for a draft equal to the mean) is concerned, we should adopt the absolute maximum whether it is positive or negative. For the case of a draft less than the mean, on the other hand, negative deviations only are significant. For practical purposes, therefore, the deviation-curve should be constructed with reference to negative deviations only.

Application of the Deviation-Curve

In the ideal case of a draft exactly equal to the mean, the range may be determined graphically as shown in Fig. 7. Given the deviation-curve AB, we can draw another curve CD representing the product $N\delta$. By the construction shown in the figure, a third curve, EF, representing the ratio R/M_a can be drawn, whence R is determined. It is understood that, in the absence of knowledge of the absolute mean, the overall mean for the whole of the existing record would have to be used instead. It is of interest to note that, despite this fact, the deviation-curve will automatically indicate the maximum deviation for that mean.

If δ is expressed as in (16),

$$N\,|\delta| = N\,\frac{c}{N^m} = c\,N^{I-m} \tag{17}$$

which is the equation of curve CD. The equation of curve EF will be:

$$\frac{R}{M_a} = \frac{c}{2^{I-m}}\,N^{I-m} \tag{18}$$

For random events, m = 0.5 and, in consequence, R/M_a varies as $\sqrt{N}$, as

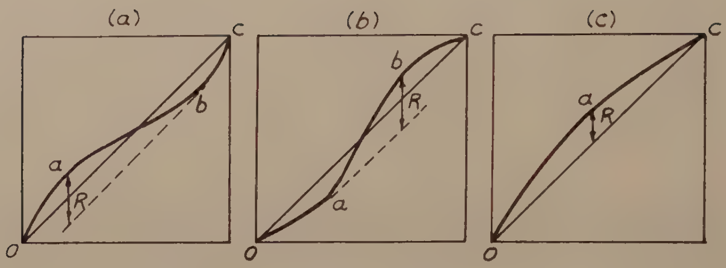


Fig. 6 - TYPES OF CRITICAL PERIODS

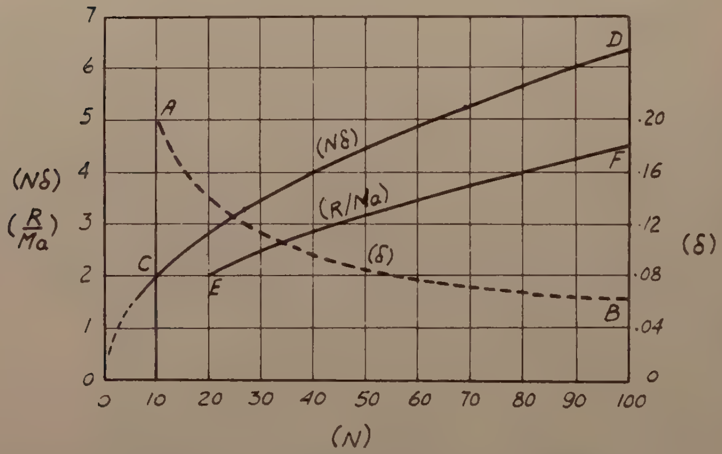


Fig. 7 - DERIVATION OF THE RANGE
FROM THE DERIVATION-CURVE

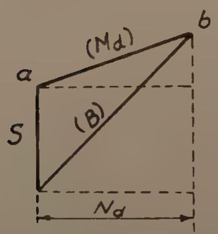


Fig. 8 - RELATION BETWEEN S, B, M_d & N_d

has already been shown. For natural phenomena, on the other hand, m would generally have some value between 0 and unity. The former value would signify that R varies linearly with N and the latter would signify that R has a limiting value independent of number (as in the case of a perfectly periodic occurrence).

Since δ cannot possibly increase with N, m cannot be negative and, according to (18), R cannot vary as N raised to a higher power than unity. The fact that in Table III such powers are encountered is due to inconstancy of the standard deviation. It is clear that if σ varied progressively with N, the exponent of N in (4) would be affected.

As regards the case of a draft less than the mean, it has already been pointed out that there is no justification for supposing the existence of a fixed relation between the storage needed in that case and the ideal range. It may be further noted that the crucial question in that case would not be the maximum deficit within a given period but the maximum deficit with respect to the reduced draft B that might be encountered at any time.

In general we have:

$$S = N_d (B - M_d) \tag{19}$$

where N_d is the length of the maximum-deficit period and M_d is the corresponding mean. This relation is demonstrated schematically in Fig. 8. Putting β = relative departure of B from M_a and δ_d = relative departure of M_d, (19) reduces to:

$$\frac{S}{M_a} = N_d (\delta_d - \beta) \tag{20}$$

Evidently, the value of S or S/M_a we should seek is that for which $(N\delta - N\beta)$ is a maximum. Now, according to (17),

$$N\delta = c\, N^{1-m}$$

Differentiating (20) with respect to N and equating to 0, we get:

$$c\,(1 - m)\, N^{-m} = \beta \tag{21}$$

whence the critical value of N_d would be:

$$N_d = \left[\frac{c(1-m)}{\beta}\right]^{1/m} \tag{22}$$

By inserting that value in (20), the required value of S will be obtained.

The graphical solution of that problem is shown in Fig. 9. The $(N\delta)$-curve in that figure is the same as that given in Fig. 7. The straight line OG represents the quantity $(N\beta)$. The critical value of S/M_a is given by the maximum intercept between the two plots.

If m is very small or zero, N_d will be very large or infinite. In such a case, the choice of a suitable value of S will have to be based solely on practical considerations.

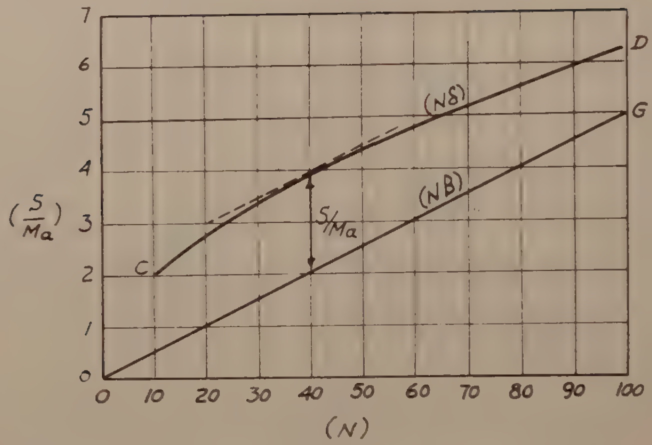


Fig. 9 - DERIVATION OF THE M/XIMUM DEFICIT

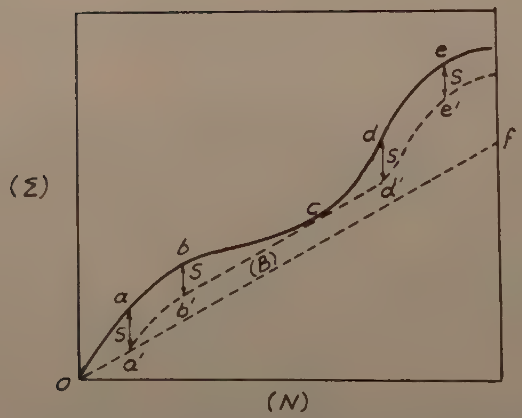


Fig. IO - REGULATION FOR A DRAFT LESS THAN THE MEAN

Program of Regulation

Having now come to the conclusion that the draft should be fixed always at some value smaller than the probable future mean, the program of regulation will be as illustrated in Fig. 10. The full line Oabcd represents the mass-curve for the supply and the broken line Oa'b'cd' represents the mass-curve for the actual draft. During such periods as ab or de, the reservoir will be full and the whole of the natural supply will have to be passed downstream. Evidently, regulation cannot be expected to proceed according to plan until the reservoir has been once completely filled. A special program will, of course, have to be laid down for the initial filling.

Case of Variable Draft

In some cases, the draft from a reservoir may have to be varied from year to year instead of being maintained at a constant figure. A case in point is that of the proposed Lake Albert Reservoir on the Upper Nile. It is known that the Main Nile receives its waters from a number of important sources besides the Equatorial Lakes, namely, the Bahr el-Ghazal, the Sobat, the Blue Nile and the Atbara. With the exception of a small fraction of the Blue Nile supply (that coming from Lake Tana) the flow from these sources cannot be put under over-year control.* As far as Egypt's requirements are concerned, there would be no sense in maintaining a constant outflow from Lake Albert. The function of the Albert Reservoir should be to make up any deficiency in the supply reaching Egypt from other sources below the requirements. Thus, the outflow from that reservoir would have to be varied between wide limits.

To determine the necessary capacity in such a case, we need a sample account of the annual requirements from the reservoir (compiled from past records of the supply from other sources) in addition to the record of the natural inflow. We should see to it, by making a proportionate reduction in the demand if necessary, that a safe margin is left between the mean draft and the mean supply.

One way of dealing with that problem is to compute the departures in the draft from its mean, transfer these departures with reversed sign to the supply and construct the deviation-curve for the latter on the basis of the modified figures. The maximum deficit could then be determined as explained in Art. 12, with reference to the mean draft.

Another way is to construct a deviation-curve for the draft separately, reduce the scale of δ in the ratio B/M_a, where B = mean draft, and plot the resultant $(N\delta)$-curve below the mean-draft line, as shown in Fig. 11. The value of S/M_a required will be the greatest intercept between that curve and the $(N\delta)$-curve for the natural supply. If a maximum is not found within a period of, say, 50 years, then the intercept at $N = 50$ may be adopted.

The second method is the more safe as it gives the worst possible combination of an excess in the demand with a deficit in the supply. The probability of such a combination may be low when the supplies from the different sources are not correlated. The question, however, is one of comparison between the additional capital outlay to be incurred and the damage that might ensue in the event of that coincidence actually taking place.

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* Unless the Aswan High-Dam Scheme, which is at present under consideration, proves feasible. In that case, the Albert Reservoir will not be needed.

Effect of Sub-Annual Fluctuations

As far as hydrological phenomena are concerned, the natural time unit of an individual observation in the consideration of long-term storage is one year. The theoretical treatment of the problem, however, would not be complete without considering the probable effects of sub-annual fluctuations on the result. It is not difficult to see that shortening of the time-unit might lead to an increase in the range of cumulated departures.

In this respect, it is to be noted that while cyclic variation in annual rainfall or run-off observations is generally erratic, practically all such phenomena are subject to more or less definite sub-annual cycles. That calls for an independent treatment of sub-annual fluctuations in either the supply or the demand.

In a given reservoir, the effects of sub-annual fluctuations are likely to be felt only when the contents are about to touch either their minimum or their maximum value as indicated by the yearly account. Some additional capacity may be needed to allow for secondary deviations within the year preceding either event. That capacity can be determined on similar lines to those followed in the design of sub-annual reservoirs.

In the majority of cases, however, the need for such a correction may be eliminated by a suitable choice of the date at which the hydraulic year is supposed to begin. If that date is so chosen that there would be no secondary filling during the last few months of the year, the reservoir contents would be most unlikely to fall below their ultimate value within the year at the end of which they are supposed to drop to their minimum. Whatever happens at the top end does not matter much since there would always be some means of disposing of excess water.

It is to be borne in mind, however, that in any work designed on probability considerations, matters cannot be expected to run exactly according to schedule. Thus, in the running of an over-year reservoir, the draft should be modified according to some sliding scale as the contents approach their prevised minimum value. That might render a highly refined theoretical treatment unnecessary.

Flood-Storage and Escapage

Since, in practice, the mean draft adopted should always be less than the predicted mean supply, the reservoir would, as a rule, have to remain for long periods on end at or near its maximum level. That is clearly demonstrated in Fig. 10. In a small reservoir, any sudden rise in the inflow during such periods might be met by instantaneous escapage through a spillway. In a reservoir of large surface area, the provision of a spillway of sufficient evacuating capacity to prevent an appreciable rise in the water level would not be practical. It might also be inadvisable to install an automatic spillway if the sudden release of large volumes of water from the reservoir is likely to cause damage in the downstream channel.

Thus, any large -capacity reservoir would, as a rule, have to be worked as a flood escape in addition to its normal function. Knowing the limiting value of the discharge that could be safely passed downstream and the maximum flood discharges (into the reservoir) to be expected, the computation of the flood-storage needed is a simple matter. For obvious reasons, the program of flood regulation should be so arranged that the relief storage of any flood be disposed of completely before the arrival of the next flood.

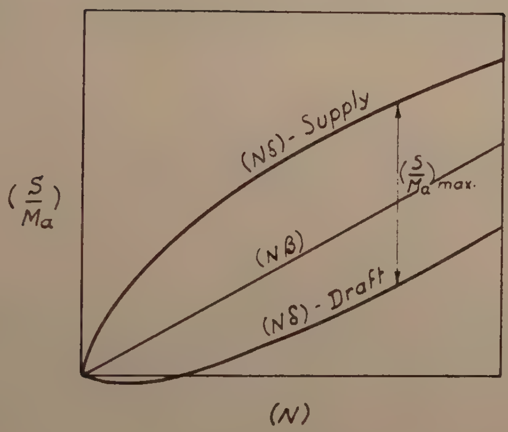

Fig. II - STORAGE FOR A VARIABLE DRAFT

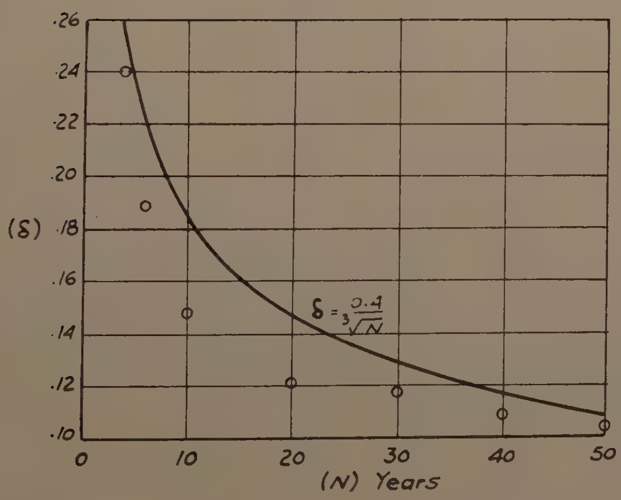

Fig. 12 - DEVIATION-CURVE FOR THE NILE AT ASWAN.

Effect of Storage Losses

When a long-term reservoir is designed so as to guarantee a certain draft, that draft would obviously include storage losses. We may write:

$$1 - \beta = \frac{B + L}{M_a} \tag{23}$$

where B here denotes the useful draft and L is the rate of loss at the adopted storage level. Hence,

$$B = M_a(1 - \beta) - L \tag{24}$$

It is clear that the closer the gross draft approaches to the mean supply, the greater will be the storage capacity needed. Since, however, storage losses also increase with the capacity, it will generally be found that there is an optimum value of the capacity beyond which B would begin to decrease again. That feature is clearly demonstrated in the practical example given in the next section.

Engineering Application

Factors Governing Design

It has been said before that in the preparation of a long-term storage project, the principal tools in the hands of the designer are sound practical judgment and a thorough understanding of the principles involved. In the foregoing treatment of the problem, the relevant physical factors have been discussed at some length in order to make these principles as clear as possible and to reveal the considerations in respect of which the designer has to use his judgment. At first glance, the method of treatment suggested might appear to be time consuming. Actually, it will be found to be more rapid than any un-guided trial and error method, in addition to giving the designer a surer footing on which to stand.

The keynote of the treatment is the Deviation-Curve. That curve gives us as complete an account as possible of the physical characteristics of the phenomenon under consideration. It is admitted that great care is needed in the preparation of that curve since it is the basic factor in the whole problem.

Once the deviation-curve has been prepared, the capacity-yield relationship for a wide range of both can be readily obtained. From relations (18), (19) and (20) of Art. 12 we get:

$$\frac{S}{M_a} = c\,m \left[\frac{c\,(1 - m)}{\beta}\right]^{\frac{1-m}{m}} \tag{25}$$

By assigning different values to β, the corresponding values of S/M_a, or S (since M_a is known), can be readily computed.

The relation between S and the rate of storage loss L must, of course, be known. A mathematical expression is not necessary, since the problem can henceforth be solved graphically. With the aid of (24), we can draw a curve

representing the variation of the effective draft B with S. On the basis of that curve, another one representing the cost-yield relationship can be obtained. The choice of capacity may then be made in full view of all the physical and economic factors involved.

Practical Example

As the present investigation was originally inspired by schemes for over-year storage in the Nile Basin, it would be natural to use one of these schemes as an illustration. The famous High-Dam scheme is well suited for our purpose. The working storage arrived at through use of the deviation-curve method, as will be seen presently, is sensibly smaller than that adopted in the officially approved design of that scheme.

The scheme involves the creation of an over-year reservoir in the valley of the Nile just upstream of the present Aswan Dam. The whole of the flow of the River, including the silt-laden flood water, will be impounded in that reservoir. To allow for loss of capacity through silt deposition, a "dead" storage of 30 milliard cubic metres is to be provided (one milliard = 10^9). The "live" or working storage, according to official accounts, is to be 70 milliards m^3. On top of that, there will be added another 30 milliards for flood relief, bringing up the gross capacity to 130 milliard m^3.

The mean natural-river supply at Aswan for the period 1871-1950 is 93 milld.m^3/annum. Deducting 4 milliards against Sudan abstractions, the most probable value of the mean supply during the next hundred years or so will be 89 milld.m^3/annum. If conservation works are carried out in the Upper Nile Basin, as projected, these conditions will change. That possibility may have been allowed for in the actual design. In the present example it will be ignored, and the figure of 89 millds. will be supposed to represent the absolute mean supply (M_a).

Estimates of reservoir areas and capacities at different levels have been prepared by the High-Dam Authority. Figures have also been given for storage losses at different levels. Evaporation losses are not difficult to estimate but it is impossible, at present, to make accurate forecasts of seepage and saturation losses. Evaporation in the Aswan district is known to be about 1000 mms. per annum. In the following computation it will be assumed that the gross rate of loss will be equivalent to a depth of 3 metres per annum on the water surface. Precipitation in the district is practically nil.

In the computation of the storage losses for different reservoir capacities, two special corrections have had to be made. Firstly, since at any adopted capacity the reservoir would not remain always at its highest level, some reduction in the corresponding rate of loss should be allowed for. Secondly, since the working storage is to be superposed on the dead storage referred to above, the corresponding loss should be reckoned on a figure 30 milld.m^3 greater.

The solution of the problem in accordance with the foregoing data is given graphically in Figs 12 and 13. Fig. 12 shows the deviation-curve for the natural supply at Aswan after correction for Sudan abstractions as mentioned above. The observed maximum deviations are well covered by the adopted curve of which the equation is:

$$\delta = \frac{0.4}{\sqrt[3]{N}} \qquad (26)$$

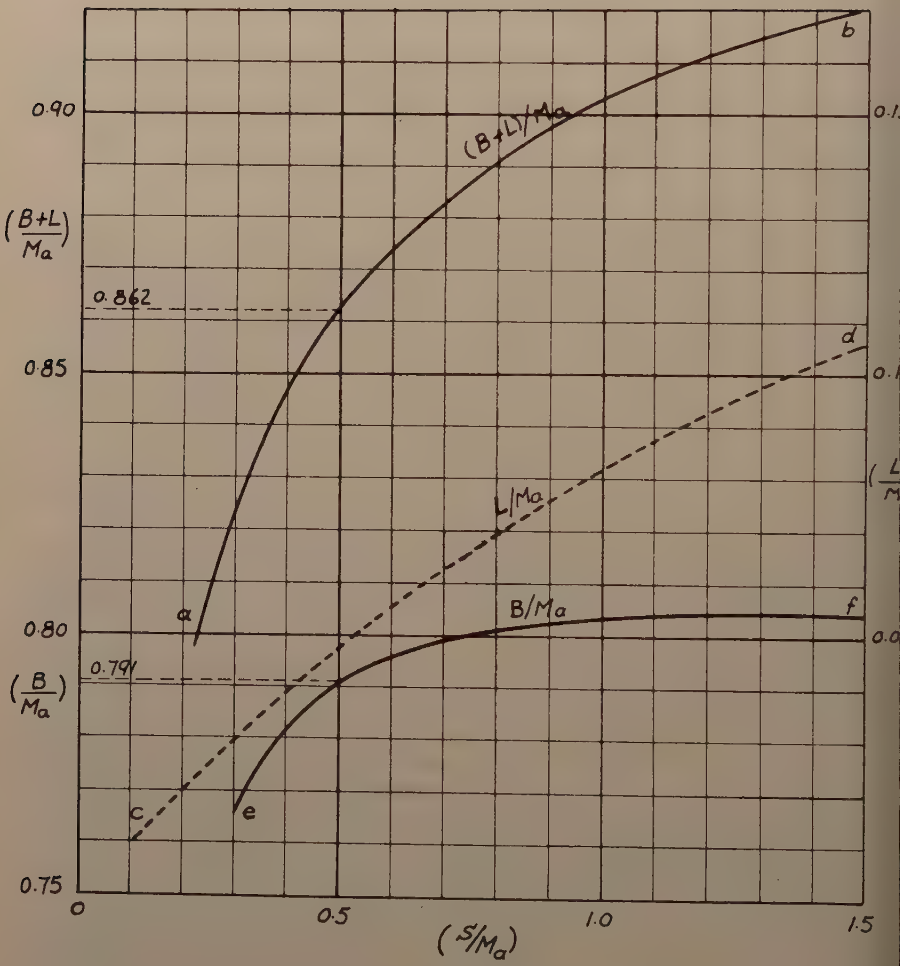

Fig. 13 - YIELD-CAPACITY CHART FOR THE ASWAN HIGH-DAM.

Thus, according to the notation used, c = 0.4 and m = 0.333.

In Fig. 13, curve (ab) is a plot of the quantity $(1-\beta)$, which is equivalent to $(B+L)/M_a$, against S/M_a. Curve (cd) is a plot of L/M_a on the same base. Curve (ef) represents the effective-draft ratio B/M_a. In the last plot we see at once that between the values of S/M_a: 0.5 and 1.0, the gain in effective draft decreases rapidly and, thereafter, B remains practically constant despite the increase in S. It is clear that if the plots were extended B would eventually decrease instead of increasing with S. The disposition and character of the B/M_a-curve evidently depend very largely on the nature of the L/M_a-curve. That calls for very careful study of the question of storage losses.

No data regarding the cost increments for different storages are at hand but it would appear from the figure that an increase in the value of S/M_a above about 0.5 would not be economical. At that value, the useful draft is 0.791 M_a, or 70 milliards m^3/annum and the working storage is 45 milliards. By raising the storage to 70 milliards (0.785 M_a) the gain in draft would be only about one milliard per annum.

The final result may be summed up in the following:

a) S = 45 milliard m^3 (gross storage = 105 millds.)
b) B = 70 milld.m^3/annum.
c) $\beta = 0.138$
d) N_d = 7.2 years.

The last figure shows that deviations for short periods may be much more important than deviations for long periods. In the construction of the deviation-curve, therefore, particular attention should be paid to short-period deviations.

In conclusion, it may be pointed out that the sensitivity of the result to the physical characteristics of the phenomenon considered, as may be gathered from the above example, renders the use of empirical capacity-formulae of a general character inadvisable.

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3. "Flood Stage Records of the River Nile." Trans. A.S.C.E., 1935. Paper No. 1944.
4. "On Mean or Average Rainfall." Proc. I.C.E., Vol. CIX (1892).
5. See "The Calculus of Observations" by Whittaker and Robinson, Art. 94.
6. Smithonean Weather Records.

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

CONTENTS

DISCUSSION
(Proc. Paper 1092)

Discussion of "TIDAL COMPUTATIONS IN SHALLOW WATER"

by J. J. Dronkers and J. C. Schönfeld
(Proc. Paper 714)

J. J. DRONKERS[1] and J. C. SCHÖNFELD.[2]—In his comment, Mr. Fenwick gives an interesting supplementary note on the question of the true simulation of the flow pattern in a distorted model. His remark applies to relatively large-scale models with moderate distortion.

In the writers' comparative discussion on models and computations, they had more particularly in mind strongly distorted, small-scale models. The following consideration will make this clear:

It is a useful principle in tidal hydraulics, to distinguish between the general tidal motion as characterized by the fluctuating levels and total flows, and more local phenomena defining the detailed distribution of the total flow. These two aspects of the tidal motion need not necessarily be treated by the same means.

In case of a tidal system of small extent, a hydraulic model on a large scale and without or with moderate distortion, may meet all requirements. In case of complicated and extensive tidal systems, however, the demands imposed upon the model from the two points of view, may be so divergent, that it is indicated to make a small-scale, strongly distorted model of the entire system for the investigation of the general tidal motion, together with one or more large-scale models of particular parts of the system for the investigation of the local problems.

The great value of the larger-scale hydraulic models for the investigation of the local tidal problems, is beyond discussion. However, for the investigation of the general tidal motion, the hydraulic model is not the only possible tool, and an electric analogue or a mathematical "model" may serve as a substitute for the small-scale hydraulic model, or as a supplementary check.

As has been stated, the comparative discussion in the paper referred in particular to hydraulic models of large tidal systems, on a small horizontal scale.

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Discussion of
"THE IMPORTANCE OF FLUVIAL MORPHOLOGY IN HYDRAULIC ENGINEERING"

by E. W. Lane
(Proc. Paper 745)

E. W. LANE,[1] M. ASCE.—The author very much appreciates the discussion of his paper by Messrs. Kuiper, Nimmo and Happ, and believes that the points they have brought out have added materially to its value.

Mr. Kuiper has emphasized the difficulties of making accurate estimates of the magnitude of the changes in streams of the nature discussed in this paper. It is true that in the present state of our knowledge, only under the most favorable conditions is it possible to make reasonably accurate quantitative estimates. However, by an application of the general principles discussed in this paper, it is frequently possible to determine positively that an undesirable action will take place. When this is the case the engineer cannot ignore it. He must make the best estimate of the magnitude and rate of change that the present state of the science and data available will permit. Although the estimates in some cases may be far from accurate, the results on the whole will be much better than if the action was entirely ignored. The engineer must, of course, be perfectly frank regarding the degree of uncertainty of his estimates. Often very rough estimates can be very valuable. For example, there is the case of a very expensive recreational development which was built at the upper end of a reservoir lake, which had only mud flats in front of it a few years after it was built. There is another case of a similar situation in which great loss would have resulted had not the plans been abandoned. In either case the result could have been foretold with very limited quantitative studies, and it is not unlikely that in the second case the plan was abandoned because such an estimate was made. As recorded observation on actions of this type increases, and the science of sediment transportation advances, more and more accurate estimates will become possible.

The writer feels that Mr. Kuiper's example presents a more pessimistic picture than occurs in many cases. The rate of aggradation fifty miles upstream from a reservoir as posed in his problem, would depend largely on the rate at which the reservoir filled with sediment, and this would probably be controlled largely by the suspended sediment, the load of which often can be measured with reasonable accuracy.

In other problems of the nature under consideration the bed material load is often the main factor in causing the changes. Its magnitude can frequently be estimated with reasonable accuracy from suspended sediment measurements, and a "rating curve" of this load for the various discharges obtained. By combining this curve with a discharge duration curve of the stream at that point, the average annual bed material load can be computed. Approximate estimates of the volume of deposition after a certain period can often be made

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by drawing profiles representing the nature of the changes from the original profile which one would expect from the figures presented in the author's paper. Computed backwater curves can be used to indicate the upper limit of the effect. The volumes enclosed between this profile and the original one can be computed, and the probable time required to make this change computed from the relation of this volume to the average annual load. The accuracy of such estimates depends to a large extent on the knowledge and experience of the one making them, but they are likely to produce much more satisfactory results than would result from ignoring the changes. In drawing such an altered profile, a study of the changes which occurred in other cases is very helpful, and the author would like to emphasize Mr. Kuiper's statement of the value of systematically collecting data on river regime and the effect of artificial interference with it as a means of perfection of the accuracy of future estimates of the magnitude of such effects.

Mr. Kuiper has correctly pointed out the difficulty, under present conditions, of obtaining accurate results by the direct application of sediment transportation formulae to these problems. The author feels, however, that if Mr. Kuiper presented in detail the results of the excellent work that was done in analyzing a problem of this general nature by Mr. Kuiper and colleagues in connection with a reclamation project on the Saskatchewan River in Canada, the readers would agree that more reliable solutions were possible in some cases than were indicated by Mr. Kuiper's discussion.

Mr. Nimmo has correctly called attention to the effect of attrition and selective deposition on the shape of stream profiles. The author did not include this action in his paper since it would have unnecessarily complicated the presentation of the ideas he wished to introduce. The magnitude of this effect can usually be obtained from the shape of the stream profile in its initial condition, and the changes of the nature considered in this paper can be treated as changes from this original profile.

Attrition and selective deposition are important aspects of geology. By observing the gradual decrease in particle size with distance in sediments laid down many millions of years ago, geologists can show that mountain ranges formerly existed where little evidence of them is present today. Experiments show that attrition is appreciable in the transport of sediment of large sizes, but is very small for material of sand size.* In a given stream it would be difficult to separate the effects of attrition and selective deposition. In general it is probable that where the valley is narrow, so that the opportunity for storage of sediment is limited, the attrition is the predominant action, but where large storage spaces are available, selective deposition is the major effect. Attrition has been extensively studied** and

* C. C. Inglis and D. V. Joglekar—Rate of Sand Attrition in Channels, Indian Waterways Experiment Station, Poona Publication 8 pp 62-64, 1945.

** S. Shulits—Fluvial Morphology in Terms of Slope, Abrasion and Bed Load—American Geophysical Union Transactions, Vol 17, Part 2, pp 440-444, July 1936.

S. Shulits—Rational Equation of River Bed Profile—American Geophysical Union Transactions, Vol. 22, Part 3, pp 622-630, 1941.

W. C. Krumbein—The Effect of Abrasion on the Size, Shape and Roundness of Rock Fragments—Jour. Geol. Vol. 49 No. 5, pp 482-520, July/Aug. 1941.

A. O. Woodford—Stream Gradients and Monterey Sea Valley, Bull. Geol. Soc. of America, Vol. 62, pp 799-852, 1951.

separation of the two effects in any case could probably best be made by computing the attrition effects and assuming that the remainder of the change was due to selective deposition.

The picture and description of Coopers Creek in South West Queensland presented by Mr. Nimmo is very striking. It probably represents an extreme case of braiding in a channel, due to aggradation, where the spreading of the channels over a very large width was permitted.

Dr. Happ's discussion is valuable in illustrating the difference in the thinking of the geologist and the engineer due to the relatively minute time involved in the cases handled by the engineer when compared with those usually considered by the geologist. He is right in calling attention to the fact that the classification of streams using the terms youth, mature and old age is only descriptive, but the use of these terms descriptively by engineers would be useful in many cases. Dr. Happ is also correct in noting that stream flow variation is a factor in many morphological problems encountered by the engineer. The effect of variable flows as compared with uniform ones, is that the variable flows transport higher sediment loads for the same total quantity of water involved, than uniform flows. The effect of increasing or decreasing the variability of flow, would have the same effects in the stream action as increasing or decreasing respectively, the discharge of the stream.

The author's classification of the morphological changes of a stream was drawn up to present the cases which are likely to be encountered by the engineer in a way which would be readily grasped by him. Other classifications are possible, and for a pure science or geological approach, perhaps Dr. Happs classification would be better. His classification could be reduced to six by combining the two items under the same letter. For example, the two items designated B could be combined into one stated: change of slope caused by lowering or rise of the base level. The author's six cases could similarly be reduced to three.

For engineers, the principal objection to Dr. Happ's classification is his inclusion diastrophic uplift or crustal tilting. The hydraulic engineer continually encounters cases where crustal tilting which has taken place in the past is important, but the author knows of no case where the crustal tilting which may occur during the life of an engineering project has to be taken into account in its design.

Dr. Happ has called attention to the use by geologists of a second concept of grading as applied to streams which differs fundamentally from the equilibrium concept discussed by the author. This second concept involves smoothing out of the stream profile and is not concerned with equilibrium or balance. The author's treatment largely followed the balance idea, which is presented by Kesseli(21)* and Mackin.(25) The smoothing out concept of grading is probably not widely used since the author did not find it in his rather extensive search of geological literature. Since reading Dr. Happ's discussion he has encountered a possible case of it in the definition of "graded" by L. La Forge given in A Glossary of the Mining Mineral Industry.**

Graded is defined by La Forge as "Brought to or established at grade, through the action of running water carrying a load of sediment, by eroding or degrading at some places and depositing or aggrading in other places."

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* These references refer to the original list.

** U. S. Bureau of Mines Bulletin No. 95—A. H. Fay.

This eroding at some places and depositing at others may have involved in La Forge's mind a smoothing out of the profile, but this is questionable since his definition of "grade" is "That slope of the bed of a stream, or surface over which water flows, upon which the current can transport its load without eroding or depositing."

In view of the two different concepts of grading held by geologists, Dr. Happ is probably correct in suggesting that engineers use the term "poised" suggested by G. H. Mattlies Hon. MASCE(16) rather than graded, when designating a stream in balance. Another term to describe this condition which may be useful is "quasi-equilibrium," as used by Leopold and Miller.* However, regardless of what is the best terminology, the engineer is indebted to geologists for much valuable work in this field.

Since writing the original paper, it has occurred to the author that a better way than the "equation" $Q_s d \sim Q_w S$ to present the relations between the factors involved would be to use an analogy to a balance, in one pan of which was Q_s and d and in the other pan Q_w and S, as shown in Fig. 1. To reestablish an equilibrium which existed in a stream between these four factors Q_s, d, Q_w and S when it was upset by a change in any one of them, it would be necessary to make a change in the opposite direction in the factor in the same pan as the changed factor causing the disequilibrium or change in the same direction as the disturbing factor in one or both of the factors in the other pan, or a combination of these three changes. These changes are of the same pattern as would have to be made in the analogous case of a balance in Fig. 1 if the variables were considered to represent weights. For example, an equilibrium which was upset by an increase in Q_s could be restored by a decrease in the values of the factor d which is in the same balance pan or an increase in either or both the factors Q_w or S which are in the other pan, or a combination of decrease in d and increase in Q_w and/or S.

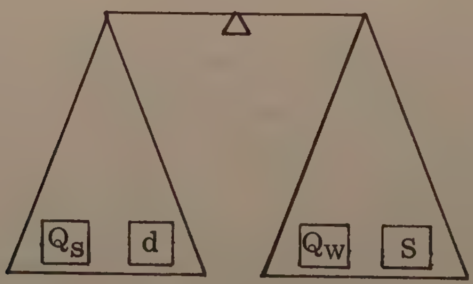

Fig. 1

* L. B. Leopold and J. P. Miller—Ephemeral Streams—Hydraulic Factors and their Relation to the Drainage Net, U. S. Geological Survey Professional Paper 282-A, 1956.

Discussion of "MINIMUM PRESSURES IN RECTANGULAR BENDS"

by M. B. McPherson and H. S. Strausser
(Proc. Paper 747)

M. B. MC PHERSON,[1] A.M. ASCE and H. S. STRAUSSER,[2] J.M. ASCE.—The authors are indebted to Professors Shukry, Robertson and Rice for the supplemental information and clarification given in their discussions. Their unanimous endorsement of the method presented for predicting minimum pressures was particularly heartening.

It had been hoped that the discussion would be limited to closed rectangular bends. However, Professor Shukry presented some valuable information related to subcritical open channel flow, and Professor Robertson presented hitherto unavailable limiting Reynolds number data for elbow meters. The suggestion by Professor Rice on the use of vanes to minimize cavitation deserves serious consideration by both designers and researchers alike.

Concerning effects of viscosity, the lowest Reynolds number in tests performed on the Waynesboro and Mt. Alto models was approximately 10^5. Because of limitations in size or velocity dictated by components other than closed bends in model studies of hydraulic structures, it appears that data for values less than 10^5 would seldom be encountered. If the data of Figure A by Robertson is applicable as well to closed bends of rectangular cross-section, the lowest value for which such model tests on bends should be run would obviously be 10^5. Unfortunately, the authors do not have substantiating data. The data of Figure 2 by Shukry are all for Reynolds numbers less than 10^5.

The paper was directed towards design rather than analysis. However, the corrections detailed by Shukry might provide a better appraisal of model results, particularly for low values of x.

Robertson has emphasized the point that correlation with irrotational flow piezometric head, as presented, is limited to the station of the maximum piezometric head differential. Put another way, it would appear that the general curvature of the streamlines in a closed rectangular bend are closest to the curvature of the bend itself at a station about 45^0 from the P.C. Some streamline curvature of necessity extends beyond the bend curvature, and at every station other than that of closest coincidence, is less than the bend curvature. This apparent coincidence of streamline curvature at the station of lowest head is the only justification for using a frictionless approach in evaluating minimum pressures.

Irrotational streamline curvature, as reflected in corresponding differences in head, may be obtained analytically, graphically or by analogy. The over-all exact analytical solution is beyond the mathematical capacity of most engineers, including the authors. Graphical solution by means of a two-dimensional flow net is entirely too inexact, particularly for bends of short radii. At the conclusion of some studies on irrotational flow through bends of

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circular cross-section by the method of electrolytic-tank analogy, Murthy[3] recently performed a similar test on the Waynesboro model bend. The plastic bend, with one side removed, was filled to mid-width with electrolyte, and electrical potential drops were measured. The data obtained for the boundaries appears in Figure 5a, while the apparatus used is shown in Figure 6, since it may be of interest to some readers. For the analogy data, h_u is the equivalent piezometric head before or after the bend in the zones of uniform (parallel streamline) flow. The rearranged data from Figure 5 of the paper is also shown, for comparison. Inasmuch as the total energy in the latter case decreases with distance, an equitable comparison is obtained by using a varying reference; the term h_e is the piezometric head as previously recommended for use; that is, $V^2/2g$ below the total energy line shown dashed in Figure 5. Note that any irregularities present in the hydraulic model were reproduced per se in the analogue. The value of C_k obtained from the analogue was 1.50 vs. 1.45 for Figure 5 (1.415 in Table 1 of the paper was the average value of several runs with a variety of approach conditions) and 1.43 from Equation 2. Despite probable inaccuracies due to construction and measurement, a reasonable comparison is obtained. Considering the inherent secondary motion, varying boundary shear and probable separation, it is indeed surprising that there is any semblance of correlation to irrotational motion. According to Figure 5a, the piezometric head distribution for a closed rectangular bend is quite similar to that for irrotational flow. The argument offered by Robertson concerning the proper limited use of potential theory is valid only with regard to a station having similar streamline and boundary curvature. The "free-vortex" is a limiting case with streamlines forming concentric circles.

In design, the use of Equation A set forth by Robertson yields lower values of C_k and hence lesser anticipated minimum pressures than Equation 2 of the paper, regardless of simplicity. Hence Equation A cannot be regarded as conservative for use with rectangular bends, especially for low values of x.

Since 90° circular bends have been introduced to the discussion, some attention must be given to the differences in results obtained for both types of bends. The available data presented in Figure 2 of the paper shows both consistency and correlation with Equation 2 for the 45° station of rectangular bends of 90° and 180° central angle. Similar data restricted to 90° circular bends at the 45° station (Figure 2 of Reference 2 of the paper) show considerable scatter, with representative data as often approaching Equation 2 as A, and with some data yielding values of C_k well in excess of those for Equation 2. From this limited evidence, the use of Equation 2 for predicting safe circular bend performance could hardly be considered conservative, much less Equation A. The following Table includes data from two elbow meters tested with water at Lehigh, together with data by Murthy from half-section electrical analogy measurements on the same elbows (these two elbows were used since flow data gave unusually high values of C_k for given x than indicated by most of available data):

3. "Potential Flow in 90° Bends by Electrical Analogy," by D. S. N. Murthy, May, 1956, part of M.S. program, Lehigh University Library.

Tabulation of Values of C_k at 45°

for 90° Circular Elbows

	x = 3.50	x - 4.50
From flow test -	1.58	1.00
By Murthy, electrical analogy -	1.55	1.02
Using Equation 2 -	1.28	0.95
Using Equation A -	1.14	0.89

The data obtained by electrical analogy, above, should be a reasonable approximation of the irrotational case. Only three bends (the highest values of x) out of six tested at Lehigh gave values surpassing those for Equation 2. The C_k values for all six exceeded Equation A, with x = 2.25 to 4.5. Therefore, with circular bends, the use of Equation 2 for design calculations does not appear to be conservative. Further, it is evident that a corresponding equation based upon irrotational flow in a circular bend would differ appreciably from Equation 2 and even more from Equation A. The mathematical determination of the irrotational field in a bend of circular section is more difficult than that for a rectangular section, if it is solvable at all. As far as results are concerned, it would appear that the points of separation in a rectangular bend must be relatively fixed, or close to the 45° station. The variety of results obtained with circular bends, on the other hand, might conceivably arise from a more variable point of separation, or some other factor not readily discernible. The authors have observed a relatively greater effect on the head distribution as the result of upstream disturbances on circular bends than on rectangular bends.

From the available evidence, the design criteria appears to be more definite for rectangular bends, due to an apparent lack of sensitivity.

The experiments by Mr. Murthy were sponsored by the Civil Engineering Department and conducted in Fritz Engineering Laboratory, Lehigh University.

RECTANGULAR BEND - WAYNESBORO

θ = 90°

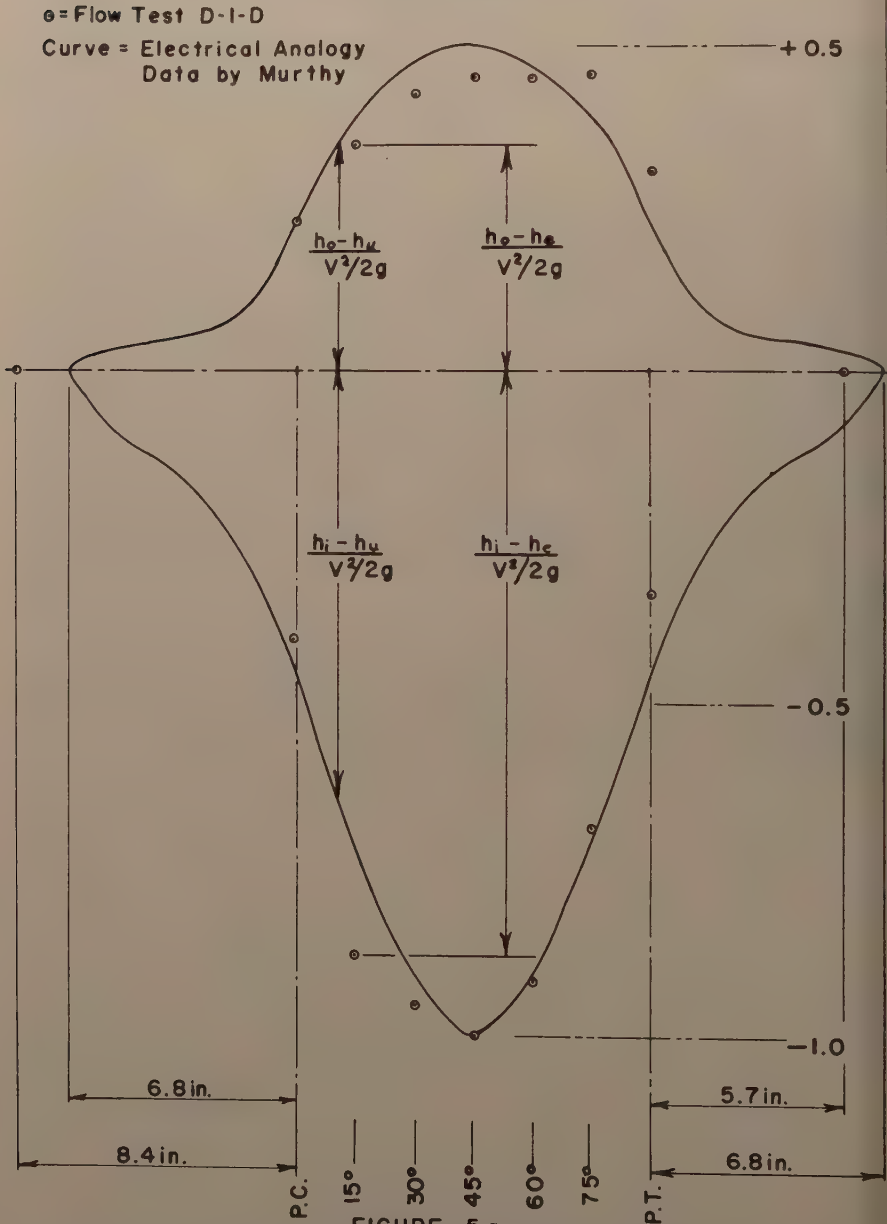

FIGURE 5a

ELECTRICAL ANALOGY APPARATUS
(The Waynesboro bend is in the foreground)

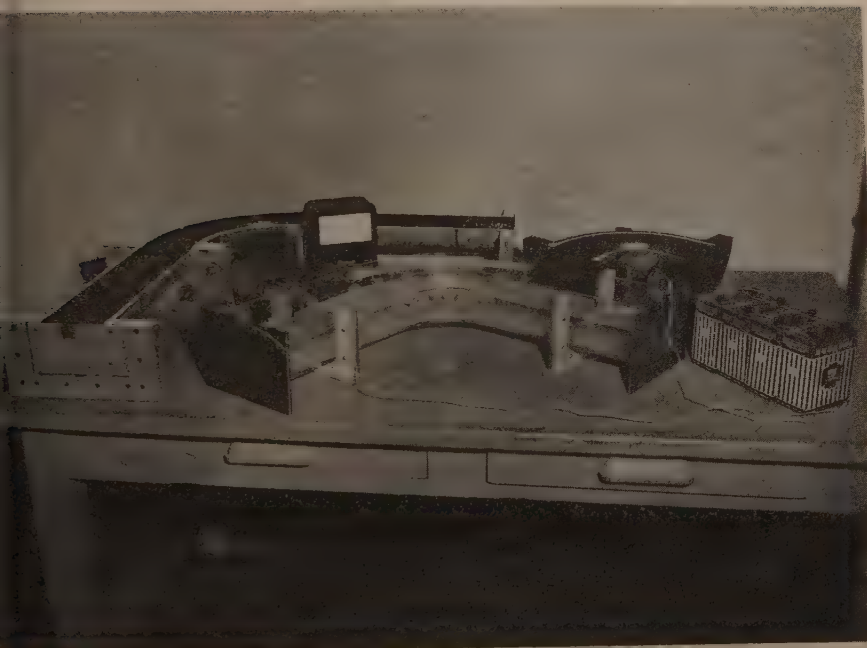

Figure 6

Discussion of
"RAINFALL DEPTH-DURATION RELATIONSHIPS"

by Herbert M. Corn
(Proc. Paper 840)

HERBERT M. CORN,[1] A.M. ASCE.—The discussions by Messrs. Hershfield and Wilson primarily question whether the storms of the United States (Yarnell's tabulation of the most intense storms),[2] which were used in the studies of rainfall regimes of French Morocco, were analogous and representative. Further, questions are raised regarding the basis of extending the mean percental depth-duration curve, derived from the studies, beyond the range of data used for the formulation. Actually, both questions are pertinent to possible future practical utilization of the curve, but neither questionable practice was applied directly to the problem nor intended for use in that manner, without proper modification for regional differences. It is fundamental that areas with a high incidence of thunderstorms or having low annual precipitation totals, whichever is characteristic, would not be used to determine regimes of other meteorologically dissimilar areas.

As noted in the subject paper, the studies described therein were concerned principally with determining the existence of probabilities in the logical appearance and sequence of incremental maximum subintensities within rainfalls, regardless of physical or geographical origin. It was desirous to apply the conclusions derived from the studies to the problem at hand, when appropriate methods for their application had been established. This, in fact, was accomplished, in Morocco, using rainfall records collected by the local weather observation stations and government experimental farms. Unfortunately, the background data, used at that time for the analysis, is not in the possession of the author for presentation at the present, however, the results therefrom do compare favorably with later frequency-intensity-duration analysis prepared by the Cooperative Studies Section, Hydrologic Services Division, Weather Bureau, U. S. Department of Commerce.[3] The latter studies indicated the 2-year one-hour rainfall, for two particular airbase locations in Morocco, to be 0.60 inches for one and 0.50 inches for the other. The 2-year one-hour rainfalls that were determined for the same locations, by utilizing 6-hour local data and interpolating therefrom by use of the curve developed during the studies of short-time precipitation from stations in the United States, were 0.80 inches and 0.65 inches respectively. The depths deduced by both studies are favorably comparable and certainly within the realm of practical accuracy.

1. Project Engr., Porter-Urquhart, McCreary & O'Brien, Cons. Engrs., Newark, N. J.
2. "Rainfall Intensity-Frequency Data," by D. L. Yarnell, Misc. Publication No. 204, U.S.D.A., 1935, pp. 9-23.
3. "Rainfall Intensities For Local Drainage Design in Coastal Regions of North Africa, Longitude 11° W to 14° E. For Durations of 5 to 240 minutes and 2-, 5-, and 10-year Return Periods," U. S. Weather Bureau, Dept. of Commerce, Washington, D. C., 1954.

The author is engaged, at the present time, in hydrological studies in conjunction with a highway design program in Honduras, Central America. Intensity-duration-frequency curves are being prepared for the project by utilizing the methods of synthetic development previously used successfully in North Africa. It is proposed to describe the methodology and submit the background data and analysis to ASCE for publication upon its completion in the near future.

The assumption, by the discussors, that the storms selected from Mr. Yarnell's studies covered only the region of Central United States is not entirely correct. Actually, the data used, for the analysis, were the first sixty most-intense rain storms tabulated in the publication[2] and they are representative of geographical areas covering thirty-six states from Eureka, California to Eastport, Maine and from Baker, Oregon to Apalachicola, Florida. Therefore, the thunderstorm-day, dewpoint, or 2-year, 24-hour precipitation comparisons of Central United States and Morocco as noted in the discussions are inapplicable. Furthermore, as previously mentioned, it was realized that modifications of the conclusions derived from the studies would be necessary to compensate for the differences in climatic conditions, when methods for their practical application had been determined.

The sixty storms analyzed were, in fact, generally convective and mostly summer rains. Although, the selection of this type of rainfall was coincidental with its position in the tabulation by Mr. Yarnell, it was of particular interest with regard to the engineering design of drainage facilities in Morocco.

The atmospheric circulation affecting the climate of North Africa is primarily the west-east drift of the Atlantic Polar Front, which is generally located along the fortieth parallel in the winter. During this season, for most parts of Morocco, the greatest portion of the annual precipitation is accumulated. According to available meteorological information,[4] these frontal rains are rarely of long duration, being on the fringes of passing cold fronts, and usually fall in the form of showers. Torrential rains are generally not observed during the winter and the incidence of thunderstorms, in that season, is rare. However, during the summer and autumn months thunderstorms are the most prevalent form of precipitation, in the areas studied, and average generally fifteen to thirty a year. The area, wherein the incidence of thunderstorms approached the higher figure, also, was characterized by a minimum number of annual precipitative days; equalling on the average less than fifty. It follows, therefore, that the heavy rains resulting from thunderstorm activity, which often produce maximum runoff, would be of special interest to the engineer. Also, for this reason, it may be stated that a relevancy existed between the studies of the convective storms from Mr. Yarnell's tabulations and the critical storms that could be expected to occur over the project areas.

Another question raised by Messrs. Hershfield and Wilson, as it is understood by the author, was the advisability and validity of extending the mean percental curve derived from short-time precipitation data to cover longer periods, such as 12- or 24-hours. The author agrees that there is little basis for the extension of the curve beyond the limits of the foundation data, although in some cases where the application had been made, the resultant fit was, for all practical purposes, satisfactory, as shown in Figure 6 of the paper. This may be explained by the "within burst" distribution of rainfall

4. "Global Physics and Meteorology in Morocco, Report on Our Knowledge in 1947," by G. Bidault and J. Debrach, 1947.

increments, as noted by the discussors. Similar relationships were found in an analysis[5] of long duration continuous rainfalls, although obviously the relationships will not apply to "among burst" distribution. At the present time, the studies of more than 160 long-duration storms, for time periods of from 6-hours to 6-days, have been completed which conclude that similar probability relationships exist for the rainfalls examined although different quantitative curve definitions were obtained, as had been expected. It is believed these studies corroborate the conclusions of the studies of short-time precipitation described in the paper.

No assumption was made by the author regarding the representation of the conclusions from the analysis of short-time precipitation to 24-hour, or longer rainfall regimes of remote areas. The discussors' comments on the periods of "no-rainfall" for a normal 24-hour time period is correct, but to carry the thought to an extreme, comparisons of this type could be made between one-day, monthly and annual rainfalls, which would be unrealistic. The author is aware of previous studies made by Messrs. Jarvis[6] and Linsley[7] and the Corps of Engineers regarding the uniformity of percental relationships of storm rainfall and annual totals for certain areas, but did not intend to imply the generalization of the mean curve derived from the studies of short-time precipitation to an application of this magnitude.

The intent of the curves of Figures 3 and 4 of the paper have been apparently misunderstood, somewhat, by the discussors with reference to the similarity and the 1- to 24-hour rainfalls, although the ratio has general application. The curves of Figures 3 and 4 indicate that for continuous ("within burst") precipitation the ratio of the maximum 5-minutes to a total 120-minute storm will be the same as the maximum 1-hour to a total 24-hour storm. This general application is evident from examination of Table 2 and Figure 6 of the paper. The comparisons of 1- to 24-hour rainfalls of Central and the entire United States, which were cited, is predicated upon regional climatic differences of the areas and the lack of continuity of precipitation, which is normal to the longer periods as noted in the discussion.

Figure 8, of the discussion, showed the regional differences in the 10- and 60-minute rainfalls, between Zones I and II, of western United States. The mean ratio of the 10-minute rainfalls to 50-minute rainfalls, based upon the summating and averaging of ratios for both zones, is approximately 0.36. The ratio for these periods as determined by the curve shown in Figure 4, of subject paper, is 0.35. Further, the mean ratio for the period in question, as shown by the curves of Figure 9 of the discussion which had been mentioned in the paper for reference[8] is approximately 0.38. Consequently, it is believed the comparisons herein noted are reasonable evidence of the probability in the uniformity of intensity-duration relationships expounded in the paper.

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5. "A Study of Rainfall Depth-Duration Relationships," unpublished thesis for Master of Engineering Degree by Capt. C. J. Cox, Agric. and Mech. College of Texas, May, 1955.
6. "Rainfall Characteristics and Their Relation to Soils and Runoff," Trans. A.S.C.E., Vol. 95, 1931, pp. 379-423.
7. "Frequency and Seasonal Distribution of Precipitation over Large Areas," Trans. A.G.U., Vol. 28, No. 3, June 1947, pp. 445-450.
8. G. R. Williams, Hydrology, Engineering Hydraulics, P. 275.

The illustration by the discussors of the limitations of extrapolation of the curve beyond the range of data is noteworthy and is an excellent example of a common error committed by the uninitiated. The misuse of empirical formulae is commonplace and the author concurs that conclusions derived in this manner may not only be misleading but often can result in extravagance of design or worse, the failure of hydraulic structures.

Unfortunately the regions, where the synthesis of maximum incremental periods would be appropriate, are typified by lack of detailed rainfall data. However, it has been the experience of the author that the technicians maintaining the observation stations in these areas are rarely unable, after years of experience recording data, to satisfactorily bracket the durations of particular rainfalls by mannerisms of notations or remarks noted in the logs. For instance, in Honduras the accumulated rainfall depths are logged at the end of each hour and in most cases notations are made regarding the estimated portion of the preceding hour within which the rain occurred. This is particularly true of heavier rainfalls. Obviously, information of this type cannot approach the accuracy of mechanically recorded data. However, until the latter becomes available, a means of deriving intensity-duration relationships is possible and practical for use in these areas.

The dissimilarity of measurements for stations of rather close proximity and the inconsistencies of data for the same station was a condition that generally existed in most locations from which data was available and examined in Morocco. The constancy of its appearance precluded the possibilities of their being associated with convective precipitation only. It is noteworthy to mention that the Cooperative Studies Section, Hydrologic Services Division, Weather Bureau, U. S. Department of Commerce also found this condition to be troublesome in their studies in the same region.

The storm of June 21, 1933, at Burlington, Vermont, was considered to be a peculiarity, by the author, inasmuch as the behaviour of the maximum incremental periods, of this particular storm, was entirely different than the patterns of all the other storms used in the analysis and comparisons thereafter. The fact that storms of these types regularly occur at Burlington does not increase its weighted value in a probability analysis when compared to the weighted averages and normal similarities of all the other storms examined.

The author is grateful and appreciative for the response and comments of the discussors and hopes the paper and discussions serve as a small step in the direction of closer association between the meteorologist and hydrologist.

Discussion of
"RESEARCH NEEDS IN SEDIMENT HYDRAULICS"

by Enos J. Carlson and Carl R. Miller
(Proc. Paper 953)

ARTHUR I. MC CUTCHAN,[1] A.M. ASCE.—Under the heading "Measurement, Standard Samplers," reference is made to the limitation of standard samplers in obtaining suspended material near the bed of a stream. In the authors' reference 2, it is pointed out that the US D-49 sampler has a limitation at the other end of the range, in that it is basically unsuitable for sampling streams more than about 20 feet deep with two-way sampling, or 40 feet with one way sampling. Where rivers may in flood reach a greater depth than this, the need for sampling at these high stages is accentuated by the fact that, as shown very clearly in the authors' figure 4, sediment concentration tends to increase with discharge, so that the sediment load carried during flood is an even higher percentage of the annual load than is the flood discharge compared with annual stream flow, i.e. if it is important in stream flow studies to have accurate measurement of flood discharge, it is generally even more important in sediment studies to measure sediment load during floods.

This depth limitation of the US D-49 sampler is the main difficulty that has arisen in the small amount of work that has been done in Queensland. On the Dawson River where sampling was attempted in February, 1956, with a D-49 sampler modified to take Imperial pint milk bottles, the river rose to a near-record height of 72 feet, and in 12 days the stream discharge was about 1,800,000 acre feet compared with the mean annual discharge of about 400,000 acre feet.

In an endeavour to overcome this difficulty without going to the complication of the US P-46 sampler, an attempt was made to use a flexible plastic bag inside a perforated rigid case which could be used in the modified D-49 sampler. This approach is suggested in the authors' reference 2, but no details are given. The sampler, which had been obtained on loan from the Snowy Mountains Hydro-electric Authority, had to be returned before tests could be completed, and the work was carried out only in a small laboratory flume, without any field testing. However, from a comparison of volume of sample obtained with the flexible container and with the milk bottle under the same conditions, it appeared that satisfactory results could be obtained in stream velocities not less than about 4 feet per second. Presumably the decline in volume obtained in the plastic container at lower velocities is due to the increasing proportion of the velocity head of the water that is used in opening the bag. Best results were obtained with the air outlet plugged. The material used for the bag was a poly-ethyl plastic 2 inch thick.

The main advantage of this type of container is that it enables point sampling to be carried out by attaching to the nozzle of a D-49 sampler some simple type of mechanism as illustrated in the authors' figure 2A, although the

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1. Executive Engr., Irrig. and Water Supply Comm., Queensland, Australia.

operation of the flap would be more difficult than on the hand sampler. Sudden inrush effects are avoided without the complication of special provision for balancing pressures at all depths as is required with the rigid container.

Reference to the results of research along these lines should be of considerable interest.

SAM SHULITS,[1] M. ASCE.—The authors present an excellent summary of what is known and what should be known in sediment hydraulics. The need for short-range quantitative prediction methods, say for 50 to 100 years, is emphasized. As the authors demonstrate, this is the realm in which the hydraulic engineer or the sedimentation specialist has achieved such remarkable progress- since the middle thirties when E. W. Lane and the writer in the Bureau of Reclamation ventured to predict the morphologic effects of Hoover Dam (then the Boulder Dam) on the Colorado River.

The twelve "phases of sediment hydraulics wherein research is needed," summarized by the authors on pages 953-18 to 19, point to other relevant research urgently needed, and described below.

Fact-Finding Surveys

The topic, "bed material transport computations," is such a corner stone to the whole field of sediment hydraulics, that it is surprising that no authoritative and definitive comparative study exists to delimit the utility of the numerous formulas for the bed-load or the unmeasured sediment transport. Though two new formulas or computation procedures at least, have been added in the last few years, no one has come forth to defend, condemn or relate the 14 formulas available at the last count of the writer. Experienced and capable sediment hydraulicians agree neither on the best formulas nor on the limitations of those used. A courageous fact-finding survey, with supported conclusions, would certainly fill a practical research need and give the hydraulic engineer reliable knowledge on which formula to use when, where, and under what conditions.

Fluvial Morphology

The four items, "stable canals and channels, design for stability, aggradation and degradation, and deposition in reservoirs," extremely important in themselves and the subject of many current investigations, are also facets or components of the grand theme, fluvial morphology. Bank revetments will be local and temporary makeshifts, either in creeks or large rivers, if the migration of the so-called "unstable" bend or bank is not considered. Perhaps the alleged "instability" is partly due to the tortuous lateral movement of the whole stream. The aggradation and degradation which follow cutoffs or channel rectification are part of the form adjustment of the profile caused by the drastic corrective measure. The morphology of these horizontal and vertical shifts of the stream is in need of considerable research, so that the overall plan and profile configurations can be predicted and correlated with the local corrective measures.

That such broad relationships can be found, is attested by the bold and imaginative pioneering studies of the U. S. Geological Survey, in particular, those of Leopold and Maddock,(28) Wolman,(A)* and Leopold and Miller.(B)

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* To avoid confusion in the reference numbering scheme, the writer's additions are marked A, B, etc.

Other important work on the quantitative analysis of watershed geomorphology is that of Strahler.(C,D) Wittman,(E) after studying 100 years of records for 250 miles of the Rhine River between Basle and Bingen, found a linear relation, M = KQ, wherein M is the migration of the crossings or bars in meters, K a coefficient, and Q the total discharge in cubic meters for the period under consideration. Inglis(F) has assembled empirical equations which relate river width, and the period and amplitude of river meanders. Strikingly systematic relationships for river profiles have been found by Yatsu(G) and the writer.(H,I)

Bulk versus Particle Research

The foregoing paragraph emphasizes "bulk" research, the sort in which broad and synoptic, yet quantitative laws are sought. This might be contrasted with "particle" research, (so-named for want of a better term), in which the goal or hope is to follow the behavior of the sediment particles as they are entrained into the suspended and bed load, dunes, anti-dunes, bars, banks and meanders. There exists a tendency to regard the latter class of research as "fundamental" or "basic," and even more scientific. Actually there is need for both, as the two lanes of inquiry will certainly merge eventually. With our present limited fund of design criteria, the writer believes that "bulk" research should receive more attention.

Deposition in Reservoirs

Fortunately, the distribution of sediments in reservoirs is under study at present by a Task Force of the Hydraulics Division of the Society.

Terminology

A study of nomenclature in this field may soon be necessary. The authors introduce "sediment hydraulics," Leopold and Maddock use "hydraulic geometry," while the writer likes "fluvial morphology." The term "bed-load" is awkward, as it is used to designate both the transported material and the quantity.

The U. S. Bureau of Reclamation is to be complimented on its work and progress in "sediment hydraulics," so ably described by the authors who themselves have been associated with these accomplishments.

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F. "Meanders and their Bearing on River Training," by Sir Claude Inglis, Maritime and Waterways Engineering Division, Inst. Civ. Eng., London, Session 1946-1947.

G. "On the Longitudinal Profile of the Graded River," by Eiju Yatsu, Trans. Am. Geophys. Un., vol. 36, no. 4, Aug. 1955, p. 655-663.

H. "Rational Equation of River-Bed Profile," by S. Shulits, Trans. Am. Geophys. Un., 1941, p. 622-631.

I. "Graphical Analysis of Trend Profile of a Shortened Section of River," by S. Shulits, Trans. Am. Geophys. Un., vol. 36, no. 4, Aug. 1955, p. 650-654.

Discussion of
"TRANSITION PROFILES IN NON-UNIFORM CHANNELS"

by Francis F. Escoffier
(Proc. Paper 1006)

ACHILLE LAZARD,[1]—The author's paper is extremely interesting because it permits the systematic extension of such interesting notions as those of "transition profiles" and "transition discharges."

The writer has, personally, no objection to the substitution of the word "transition" for that of "characteristic" which he had used in Ref. 1 of the paper.

In creating the notion of "characteristic depth" Mouret(2) confined himself to specifying that this depth is a characteristic of the channel bed and is independent of the discharge in contrast to the notions of normal and critical depths which do depend on the discharge.

In creating the notions of a characteristic discharge, which is the discharge for which the critical slope is equal to the slope of the channel, the writer wished to bring out the fact that this discharge separates two types of flow in the channel. The expression "transition discharge" is certainly to be preferred.

At the writer's request the author has indicated that there can exist 0, 1, or 2 transition profiles, depending on the slope of the channel.

As the water-surface profile is normally horizontal at its point of intersection with the transition profile it appears to the writer indispensable to give a name or an index to each section of the water-surface profile that can be defined. Mouret numbered the sections of water-surface profiles that he had studied with the arabic numerals 1 to 6. (These correspond to the types M_3, M_2, M_1, S_3, S_2, and S_1, in the order named, as used among American engineers.) The writer designated by the capital letters A, B, C, and C' the new sections which he had introduced. These are shown in the accompanying figure. The sections C and C' are practically horizontal because of their long inflection and appear to the writer to represent something real.

In combining certain of Mouret's sections with those of the writer, one obtains water-surface profiles very similar to those that are described in the classical manuals. However, certain of Mouret's sections do not seem to the writer to represent real water surfaces.

The author has in turn introduced new sections of water-surface profiles, some of which probably do not represent real water surfaces. It would be useful to designate the others in some convenient way.

The writer wishes finally to indicate that it is important in numerical calculations not to omit from the formulas the classical coefficient α which accounts for the unequal distribution of velocity about the mean velocity represented by V.

The coefficient α, about which a great deal is not known, should in principle vary when the cross sections and the channel slopes vary. The evaluation of this term can present difficulties.

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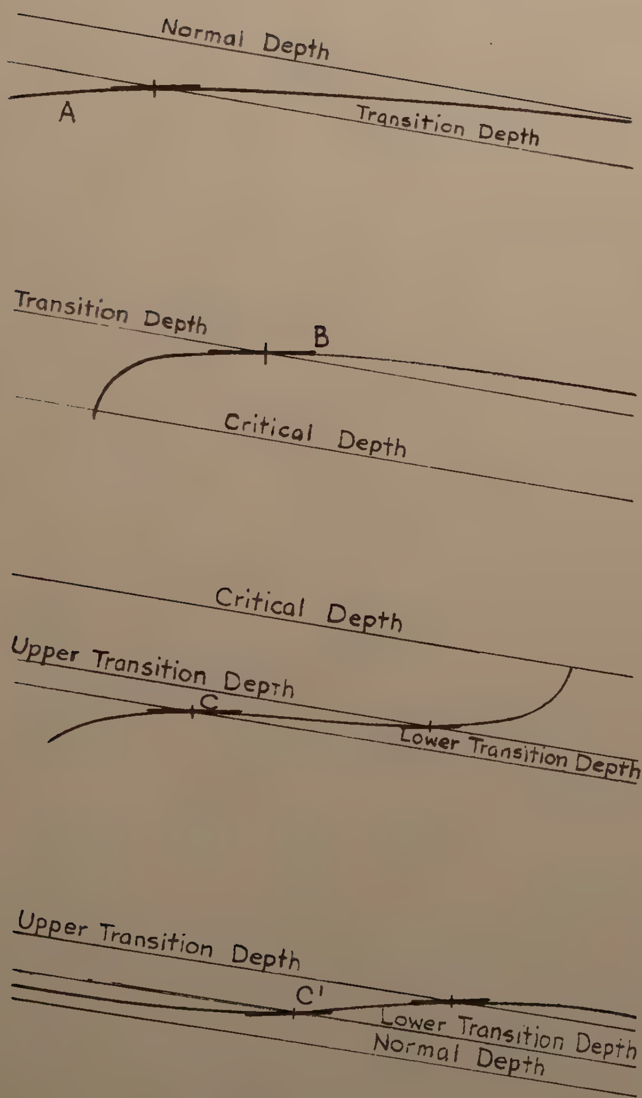
1. Ingenieur en Chef des Ponts et Chausśees, Ingénieur en Chef, French Railways, Paris, France.

In the case treated by the writer, where the channel is uniform and the channel slope is constant, a constant value could be ascribed to the coefficient α.

The introduction of this coefficient into the formulas modifies profoundly the numerical values obtained for the transition depths and discharges as these are determined by the intersections of lines that are nearly parallel. This is shown in the following table which contains the values calculated by the writer for the trapezoidal tailrace canal for the SOULOM powerhouse (Pyrenees), which has a slope of 0.004, for the extreme values for α of 1 anc 1.2.

α	Transition Depth (in cm)	Transition Discharge (in m3/sec)
1.0	90	7
1.2	45	2.

In closing, the writer will express the wish that the laboratories interest themselves in these problems and seek, by means of model experiments with canals of appropriate cross sections, to verify or disprove the theory of water-surface profiles for gradually varied flow to which the author has made such an important contribution.



TYPES OF WATER-SURFACE PROFILES

Fig. 1.

PROCEEDINGS PAPERS

ne technical papers published in the past year are identified by number below. Technical-ion sponsorship is indicated by an abbreviation at the end of each Paper Number, the ols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering anics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), ary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and ing (SU), and Waterways and Harbors (WW) divisions. Papers sponsored by the Board of tion are identified by the symbols (BD). For titles and order coupons, refer to the appro- issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were shed in Journals of the various Technical Divisions. To locate papers in the Journals, the ols after the paper numbers are followed by a numeral designating the issue of a particular al in which the paper appeared. For example, Paper 861 is identified as 861 (SM1) which ates that the paper is contained in issue 1 of the Journal of the Soil Mechanics and Founda- Division.

VOLUME 81 (1955)

OBER: 809 (ST), 810 (HW)[c], 811 (ST), 812 (ST)[c], 813 (ST)[c], 814(EM), 815(EM), 816(EM), 7(EM), 818(EM), 819(EM)[c], 820(SA), 821(SA), 822(SA)[c], 823(HW), 824(HW).

EMBER: 825(ST), 826(HY), 827(ST), 828(ST), 829(ST), 830(ST), 831(ST)[c], 832(CP), 3(CP), 834(CP), 835(CP)[c], 836(HY), 837(HY), 838(HY), 839(HY), 840(HY), 841(HY)[c].

EMBER: 842(SM), 843(SM)[c], 844(SU), 845(SU)[c], 846(SA), 847(SA), 848(SA)[c], 849(ST)[c], 0(ST), 851(ST), 852(ST), 853(ST), 854(CO), 855(CO), 856(CO)[c], 857(SU), 858(BD), 859(BD), 0(BD).

VOLUME 82 (1956)

ARY: 861(SM1), 862(SM1), 863(EM1), 864(SM1), 865(SM1), 866(SM1), 867(SM1), 868(HW1) 9(ST1), 870(EM1), 871(HW1), 872(HW1), 873(HW1), 874(HW1), 875(HW1), 876(EM1)[c], 877 W1)[c], 878(ST1)[c].

UARY: 879(CP1), 880(HY1), 881(HY1)[c], 882(HY1), 883(HY1), 884(IR1), 885(SA1), 886(CP1), 7(SA1), 888(SA1), 889(SA1), 890(SA1), 891(SA1), 892(SA1), 893(CP1), 894(CP1), 895(PO1) 6(PO1), 897(PO1), 898(PO1), 899(PO1), 900(PO1), 901(PO1), 902(AT1)[c], 903(IR1)[c], 904 O1)[c], 905(SA1)[c].

CH: 906(WW1), 907(WW1), 908(WW1), 909(WW1), 910(WW1), 911(WW1), 912(WW1), 913 W1)[c], 914(ST2), 915(ST2), 916(ST2), 917(ST2), 918(ST2), 919(ST2), 920(ST2), 921(SU1), 2(SU1), 923(SU1), 924(ST2)[c].

L: 925(WW2), 926(WW2), 927(WW2), 928(SA2), 929(SA2), 930(SA2), 931(SA2), 932(SA2)[c], 3(SM2), 934(SM2), 935(WW2), 936(WW2), 937(WW2), 938(WW2), 939(WW2), 940(SM2), 941 M2), 942(SM2)[c], 943(EM2), 944(EM2), 945(EM2), 946(EM2)[c], 947(PO2), 948(PO2), 949(PO2), 0(PO2), 951(PO2), 952(PO2)[c], 953(HY2), 954(HY2), 955(HY2)[c], 956(HY2), 957(HY2), 958 Y2), 959(PO2), 960(PO2).

961(IR2), 962(IR2), 963(CP2), 964(CP2), 965(WW3), 966(WW3), 967(WW3), 968(WW3), 969 W3), 970(ST3), 971(ST3), 972(ST3)[c], 973(ST3), 974(ST3), 975(WW3), 976(WW3), 977(IR2), 8(AT2), 979(AT2), 980(AT2), 981(IR2), 982(IR2)[c], 983(HW2), 984(HW2), 985(HW2)[c], 986(ST3), 7(AT2), 988(CP2), 989(AT2).

: 990(PO3), 991(PO3), 992(PO3), 993(PO3), 994(PO3), 995(PO3), 996(PO3), 997(PO3), 998 A3), 999(SA3), 1000(SA3), 1001(SA3), 1002(SA3), 1003(SA3)[c], 1004(HY3), 1005(HY3), 1006 Y3), 1007(HY3), 1008 (HY3), 1009 (HY3), 1010 (HY3)[c], 1011 (PO3)[c], 1012 (SA3), 1013 (SA3), 14(SA3), 1015(HY3), 1016(SA3), 1017(PO3), 1018(PO3).

: 1019(ST4), 1020(ST4), 1021(ST4), 1022(ST4), 1023 (ST4), 1024(ST4)[c], 1025(SM3), 1026 M3), 1027(SM3), 1028(SM3)[c], 1029(EM3), 1030(EM3), 1031(EM3), 1032(EM3), 1033(EM3)[c].

ST: 1034(HY4), 1035(HY4), 1036(HY4), 1037(HY4), 1038(HY4), 1039(HY4), 1040(HY4), 1(HY4)[c], 1042(PO4), 1043(PO4), 1044(PO4), 1045(PO4), 1046(PO4)[c], 1047(SA4), 1048 4)[c], 1049(SA4), 1050(SA4), 1051(SA4), 1052(HY4), 1053(SA4).

EMBER: 1054(ST5), 1055(ST5), 1056(ST5), 1057(ST5), 1058(ST5), 1059(WW4), 1060(WW4), 1(WW4), 1062(WW4), 1063(WW4), 1064(SU2), 1065(SU2), 1066(SU2)[c], 1067(ST5)[c], 1068 W4)[c], 1069(WW4).

BER: 1070(EM4), 1071(EM4), 1072(EM4), 1073(EM4), 1074(HW3), 1075(HW3), 1076(HW3), 7(HY5), 1078(SA5), 1079(SM4), 1080(SM4), 1081(SM4), 1082(HY5), 1083(SA5), 1084(SA5), 5(SA5), 1086(PO5), 1087(SA5), 1088(SA5), 1089(SA5), 1090(HW3), 1091(EM4)[c], 1092 Y5)[c], 1093(HW3)[c], 1094(PO5)[c], 1095(SM4)[c].

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scussion of several papers, grouped by Divisions.